Rationale of Single Transferable Voting System

Nita H. Shah[1], *Vishnuprasad D Thakkar*[2] *and A. K. Desai*[3]

Authors are from Department of Mathematics, Gujarat University, Ahmedabad 380009, India
E-mail:
[1]**nitahshah@gmail.com**
[2]**vd.thakkar@gmail.com**
[3]**desai_ak@yahoo.com**

In most Indian political systems, candidates getting highest number of votes get elected. Indian Parliament (Loksabha) and legislative assemblies of Indian states (Vidhansabha) elections are done using this method. It is not necessary for the candidate to get absolute majority (more than 50% votes). Similarly, in case of municipal and social association elections, multiple candidates are elected in each constituency. To ensure that in case of single single seat election, candidate having more than 50% votes and in case of multi-seat election, proportional representation is reflected in the result, we conceptualize a hypothetical system which achieves desired goal and translate the same into preferential voting system. Though the conceptualized voting system is almost impossible to implement, it leads to a system which is implementable.

AMS Subject Classification: 91B12

Keywords
Election, Preferential/ Ranked Voting, Plurality Voting, First Past The Post Voting, Indian Rajyasabha Election

1. Introduction

We elect objects or activities or persons in real life from available set of choices (candidates). Some of the examples of choices we make are:

- Place to visit
- Things to eat
- Player in the team
- Team to be member of

As an individual, we select the object or activity or place. When the choice is to be made by multiple people, we take opinon from

all the participants and decide on the basis of overall distribution of preference and select the one with maximum votes. The situation becomes more complex when multiple things are to be chosen by multiple persons. Let us take a case where for an event like sports day, sports activities are to be decided. This is a case where number of persons giving opinion (voters) are many and number of activities to be selected are more than one. Possibly informal method of election is adopted. For each activity, voting is done by raising of hand by people who prefer them and at the end, activities are listed in descending order of votes in favor for each activity and activities from top are selected.

In formal elections, voting is done using ballots as per the type of election. Description of some popular election types follow:

1.1 *Voters elect single candidate*

In this type of election, each voter gives vote to one candidate of choice. The candidate who gets maximum number of votes is elected. It is not necessary for the candidate to get more than 50% votes. This kind of election is called Plurality Voting [1] or First Past The Post (FPTP) system [2]. This is the most commonly used election type. Indian Parliament (Loksabha) and legislative assemblies of Indian states (Vidhansabha) elections are done using this method.

The Table 1 has details of percentage votes and percentage seats won by political parties in Loksabha elections - 2014. The information is based on the election results of Loksabha Elections - 2014 available on Election commission of India Website [3].

It is interesting to note that BJP has got approximately 53% seats for 31% votes; AIADMK 7.2% seats for 3.5% votes and Shivsena 3.5% seats for 2% votes. These are the cases where seat percentages are more than percentage votes. In contrast, INC gets approx. 8.5% seats for 18.5% votes; BSP no seat for 4.4% votes.

Party Name	%Vote Share	% Seats Won
Bhartiya Janta Party (BJP)	30.97	53.03
Indian National Congress (INC)	18.58	8.41
Bahujan Samaj Party (BSP)	4.31	0.00
All India Trimool Congress (AITMC)	4.08	6.65
Samajwadi Party (SP)	3.58	0.98
All India Anna Dravida Munnetra Kazhagam (AIADMK)	3.48	7.24
Communist Party of India - Marxist (CPIM)	3.45	1.76
Telugu Desham Party (TDP)	2.71	3.13
YSR Congress Party	2.69	1.76
Aam Aadmi Party (AAP)	2.12	0.78
Shiv Sena	1.97	3.52
Dravida Munnetra Kazhagam (DMK)	1.85	0.00
Nationalist Congress Party (NCP)	1.66	1.17
Rashtriya Janata Dal (RJD)	1.43	0.78
Telangana Rashtra Samithi	1.30	2.15
Janta Dal United (JDU)	1.15	0.39
Lok Janshakti Party	0.44	1.17
Others	14.21	7.05
Total	**100.00**	**100.00**

Table 1: Loksabha Elections - 2014:Votes and Seats Share

Table 2 has information on percentage votes of winning candidates in the same Indian Parliament Election 2014.

Interesting thing about the candidates getting elected is that 323 winners (63% of winners) have got less than 50% votes.

In this type of elections, two or more candidates are contesting the election where each voter selects one of the candidates and casts vote in his/ her favor. In counting of votes, votes are counted for each of the candidate and the candidate who gets the maximum votes is declared elected. It is not necessary for the candidate to secure majority votes, getting maximum votes is sufficient. The election where only two candidates are contesting election, winner has to get majority votes after ignoring NOTA and invalid

% Votes Received by Winner	Number of Winners
Upto 25 %	0
25 % to 30%	5
30 % to 35%	27
35 % to 40%	71
40 % to 45%	106
45 % to 50%	114
50 % to 55%	101
55 % to 60%	58
60 % to 66%	17
65 % to 70%	8
70 % to 75%	3
Above 75%	1
Total	**511**

Table 2: Percentage Votes for Winners

votes. For Mathematical Analysis of voting, NOTA is equivalent to *not-voting* as the vote does not go to any candidate. NOTA marks the voter as participated in voting and no other person can vote in name the same voter. Another purpose of NOTA is acceptability of candidates as a whole in voters. But for our analysis, these aspects being no relevant, we treat as not voted. In case of more candidates contesting the election, winning candidate may have less than majority (half or more) votes.

For the candidates who have won with less than 50% votes, other candidates put together have more votes than winning candidates. Had there been only one candidate against the winning candidate then possibly the result could have been different. Candidates cannot be stopped from contesting election. Let us therefore consider change in voting process.

In proposed hypothetical voting system, we allow voters to change their vote on specific condition. For this purpose, we need to trace the voters who have given vote to specific candidate. We eliminate the candidate who has got the least number of votes and

invite the voters who have voted for the eliminated candidate to give their vote to any of continuing candidates. They have choice of abstaining from this voting. At the end of this extra round of voting, the incremental votes are distributed to continuing candidates. If there are only two continuing candidates, the candidate with higher number of votes is declared as elected. If there are more than two continuing candidates, again the candidate with lowest number of votes is eliminated. Again the voters who had voted for eliminated candidate (original as well as incremental round of voting) are invited to vote again to continuing candidates. The process is continued till only two candidates survive. When only two candidates are continuing, the one with higher number of votes is declared as elected. If a candidate gets more than 50% votes at any point of time, the candidate is declared as elected without considering other continuing candidates.

Let us understand the process by an example. In the example, there are 1000 voters and 4 candidates. At any point of time, if a candidate gets more than 500 votes, the candidate is declared as elected. Table 3 has votes received by each of candidate in initial counting. No candidate gets more than 500 votes. As a result,

Candidate	**Votes**
A	350
B	320
C	250
D	80
Total	1000

Table 3: Initial Votes

as per new election model, no candidate is declared elected. The candidate with lowest number of votes gets eliminated, so candidate 'D' gets eliminated.

Voters who have voted for candidate D are invited to vote again for either A, B or C. Distribution of this voting is in Table 4

After adding incremental votes, votes against each candidate becomes as shown in Table 5. The voter, who had originally voted

Candidate	Votes
A	1
B	3
C	75
Not Voted	1
Total	80

Table 4: First Incremental Voting

to candidate D but did not vote when candidate D got eliminated, effectively the voter's vote has not gone to any candidate. So, that ballot is considered as not voted for our understanding and overall accounting of votes.

Candidate	Votes
A	351
B	323
C	325
Not Voted	1
Total	1000

Table 5: Votes after first incremental voting

At the end of first incremental voting, B gets the lowest number of votes. Now, the 323 voters who have voted for B are called for voting again. All the 323 voters will cast their votes against continuing candidates A and C. Some of them may not vote. The details of this incremental voting is given in Table 6.

Candidate	Votes
A	140
C	180
Not Voted	3
Total	323

Table 6: Second Incremental Voting

After second incremental voting, votes against continuing candidates is as per Table 7. After this transfer, candidate C has the highest number of votes, so candidate C is declared as elected.

As per original election system, candidate A becomes the winner where as after eliminating candidates B and D, candidate C gets more votes than candidates A, so candidate C becomes the winner. Candidate B and D cannot be stopped from contesting the election. However, since they have got very low vote shares, voters who had voted for them are invited to change their votes. The votes are polarized to candidate C, so candidate C becomes the winner. 4 votes marked as 'Not Voted' are the vote cast for voters to candidates who got eliminated and the voters did not turn up for subsequent level voting.

Candidate	**Votes**
A	491
C	505
Not Voted	4
Total	1000

Table 7: Votes after second incremental voting

The proposed hypothetical voting system is almost impossible to implement. Moreover, secrecy of votes is compromised. Instead of inviting voters to vote again, we let them give preference number against candidates. In the example, 80 voters who had given vote to D, will specify their second preference. As per the example, 1 voter will have A, 3 will have B and 75 will have C as their second preference. One voter has not given second preference.

In the counting process, as first step, we count votes by first preference and we get the result the same as initial votes. As next step, we eliminate candidate 'D' who has the lowest number of votes. All the ballots having preference number 1 against D are selected and are distributed by next preference. In this case, distribution will be as per Table 4 and by adding the transferred votes to respective candidates, we get votes against each candidate the same as Table 5 after first incremental voting.

Even after this transfer, no candidate emerges as winner (getting more than half the votes). So, we eliminate candidate with

the lowest number of votes. In this case candidate B is selected for elimination. All the ballots received by candidate B are distributed by next preference. Here it is important to note that for this fixed preference number is not selected. If preference number for B is p, then it goes to candidate with preference number p+1. To be precise, the vote goes to candidate having lowest preference number greater than p to continuing candidate.

If voters can give preference number to candidates rather than giving vote to one candidate then votes from losing candidate (one having lowest votes) can be transferred to candidate on the basis of next (second) preference. Some votes may go to the original winning candidate (as per single candidate voting system) and other votes may go to runners up. Again we repeat the process by transferring votes from the candidate having the lowest number of votes on the basis of next preference. After the transfer ranking of candidates may change. We go on eliminating candidates with lowest number of votes. At some point, one of the candidates may cross the 50% mark. In worst case, all the candidates except the winner and last runner up candidate may get eliminated.

We will have more details later when we discuss preferential voting.

1.2 *Voters elect multiple candidates*

In this type of election, voters vote to elect pre-defined fixed number of candidates. In this section, we will cover only the elections where preferential voting is not used. Voters give vote up to maximum number of candidates to be elected. For example, in case of municipal elections, a ward may have say 4 seats. In this case, each voter can give vote to maximum 4 candidates. This type of voting system is popular in municipal elections and social bodies (like clubs).

In most of cases, if political parties are involved then each of interested parties field as many candidates as there are seats and where parties are not involved, candidates create group called

panel of as many candidates as seats. In both the cases, they request their voters to cast votes in favor of the entire group.

1.2.1 Example

Table 8 has result of Bhavnagar Municipal Elections held in 2015. The Table is prepared from the result published on State Election Commission, Gujarat website [4].

Ward	BJP		INC		Others	
Number	% Votes	% Seats	% Votes	% Seats	% Votes	% Seats
1	46.29	25.00	50.86	75.00	2.85	0.00
2	40.49	50.00	47.32	50.00	12.19	0.00
3	42.82	25.00	49.39	75.00	7.79	0.00
4	40.76	0.00	50.26	100.00	8.98	0.00
5	54.70	100.00	41.13	0.00	4.17	0.00
6	52.73	75.00	47.27	25.00	0.00	0.00
7	51.23	75.00	46.88	25.00	1.89	0.00
8	38.45	50.00	41.00	50.00	20.55	0.00
9	45.10	25.00	54.90	75.00	0.00	0.00
10	56.30	100.00	37.69	0.00	6.01	0.00
11	68.01	100.00	30.45	0.00	1.54	0.00
12	58.76	75.00	31.77	25.00	9.47	0.00
13	65.36	100.00	26.84	0.00	7.80	0.00
Total	50.46	61.54	42.97	38.46	6.56	0.00

Table 8: Bhavnagar Municipal Corporation Elections

In ward number 1, BJP gets 25% seats for approximately 46% votes where as INC gets 75% seats for about 51% votes; in ward number 4 INC gets all 4 (100 %) seats for 50.26% votes and BJP fails to get even single seat even after getting 40.76% votes.

In this type of election, each voter is giving maximum as many votes as are seats. In case of high polarization, if 50% voters choose specific group, they would give votes to each of the candidate in the group. Net result would be each candidate of the group (voted by more than 50% voters) candidate getting more votes

and no other candidate gets as many votes as these candidates and all the candidates of this group get elected. Had this been an election where voters had been divided into as many groups as seats (restructure constituencies) and voters give single vote, result could possibly have been different. It is obvious that the candidates also get distributed. In such a situation, there is a possibility of distribution of votes for parties and winning candidate ratio may change. However, due to constitutional constraints, it is not possible to split voters. In case of social association, splitting of voters is not practical and in case of municipal elections, splitting of wards is not possible as it requires lot of administrative work.

In most of the multi-seat elections, voters give at the most as many votes as number of seats. This gives rise to different representation of different voters (as some may have give one vote where as others have given more votes) in the result. To overcome this, all the voters should give equal number of votes. We choose one vote per voter, which in existing counting process, may not result in correct representation. Like we have done process of calling voters to vote again in single seat election, we will design similar solution here which would result in correct representation.

Let us try to understand election of multiple candidates in another system of voting. In case of Gujarat Rajyasabha election 2017, 3 candidates were to be elected. At the time of election, BJP had more than half the voters. If simple voting system of giving multiple votes (3 in this case) had been used, all the 3 BJP candidates would have got more than 50% (of voters) votes and INC would have got no seats. If we go by propotional representation, BJP should get 2 seats and INC should get 1 seat. Actual result matches the propotional representaion as it is done using preferential voting using Single Transferable voting.

Similar to single seat election, let us have concept of each of winner having more votes than all the losers put together. If we eliminate candidates having lowest number of votes then there are chances that result may not have proportional representation.

For example, in case of 2 seat election and 3 candidates, most of voters (say 90%)prefer candidate number 1 and 2 where as candidate number 3 is liked by a very small group of people (say 10%). Among candidate number 1 and candidate number 2, as voters are giving single vote, it turns out that candidate number 1 gets 81% votes and candidate number 2 gets 9% votes. This can happen as voters do not know how others have voted. If we call all the voters who have voted for candidate number 1 to vote again, most of them may vote for candidate number 2. If this process is not done before elimination, then candidate number 3 who is liked by only 10% voters may get elected as candidate number 2 has less number of votes in original first round of voting. This give rise to concept of deciding threshold for winning and transferring surplus votes by secondary voting. Intuitively, we feel that this threshold should be $\left(\frac{V}{n+1}+1\right)$.

1.2.2 Mathematical Treatment

All the winners in final result should have number of votes such that each one of them has more than all losers' votes put together. In mathematical form, the equation becomes as under:

$$V = W_1 + W_2 + \cdots + W_n + L \tag{1}$$

where

V is total votes

n is number of seats (candidates to be elected)

W_i is votes for winner number i

L is total votes of all the losers put together

$W_i > L$ for every winning candidate

$$V > [(L + L + \cdots + L)\ n\ terms] + L = (n+1)L$$

As each of winner's votes are more than L, simplifying the inequality we get

$$L < \frac{V}{n+1} \tag{2}$$

From inequality 2, maximum possible value for L can be $\left(\frac{V}{n+1} - 1\right)$.

Putting this value of L in eq.(1), we get

$$V = W_1 + W_2 + \cdots + W_n + \left(\frac{V}{n+1} - 1\right) \tag{3}$$

Simplifying the equation, we get

$$\frac{nV}{n+1} + 1 = W_1 + W_2 + \cdots + W_n \tag{4}$$

Now, each of the winner has to get more than $L = \left(\frac{V}{n+1} - 1\right)$ votes. So, each of winner votes W_i has to be $\geq \frac{V}{n+1}$. Moreover, we can assume without loss of generality that W_1 has the highest number of votes. Applying the lower limit on number of votes for other $(n-1)$ winners in eq.(4), we get

$$\frac{nV}{n+1} + 1 \geq W_1 + (n-1)\frac{V}{n+1} \tag{5}$$

Simplifying this and applying condition on number of vote applicable to all winners, we get

$$\frac{V}{n+1} \leq W_1 \leq \frac{V}{n+1} + 1 \tag{6}$$

It is not possible for the candidate with highest number of votes to have $\frac{V}{n+1}$ votes as it would leave the same number of votes for each of willers and all losers put together (L).

In case of all the losers put together having highest possible number of votes, winner with the highest number of votes can have exactly $\left(\frac{V}{n+1} + 1\right)$ votes.

This magic number is called *Quota* for candidate to win.

If we consider this setup for electing only one candidate, candidate has to get at least $(\frac{V}{2} + 1)$ votes to win the election. This concept fits as extension of single seat election as explained in previous section.

To understand different possibilities of distribution of votes, we will assume that candidates and their votes are arranged in descending order of number of votes.

If each of first *n* candidates have more votes than the *Quota* then they are declared as elected. This is very unlikely in real life elections.

As the process, we will transfer winner's surplus votes to continuing candidates. If there are no more winning candidates, we will eliminate candidate from bottom. We may do batching (select more than one candidates) for elimination.

Let us build a hypothetical model like we have done in single seat election. As discussed earlier, we ask voters to give one vote. On certain condition, we will call the voters to cast their vote again to any of continuing candidates.

As voters are giving only one vote, it may so happen that one candidate from 'popular' group may get significantly more votes and other candidates from the same group may get less votes and they lose as voters are giving vote only to one without knowing how other voters voting. As a result, in contrast to single seat election, we invite voters of winning candidate to vote again to any of continuing candidate for votes exceeding *Quota*. We cannot select the voters to cast their secondary votes for desired number of (surplus) votes. We invite all voters (who have voted for winning candidate) to vote again and distribute excess votes in proportion of votes received by continuing candidates.

If no more transfers are to be done from winning candidates, we do the process of elimination of candidates having lowest number of votes.

Let us take an example, where 8 candidates from 2 parties where one party has 70% voters in their favor and 30% voters in favor of the other party. It may so happen that voter may not like all 4 candidates of a group. In our example, there are 1000 voters who have cast their votes.

Interesting thing to note here is that group 1 candidates have

Group Number	**Candidate Number**	**Number of Votes**
1	1	400
1	2	200
1	3	75
1	4	25
Total for group 1		700
2	5	120
2	6	90
2	7	60
2	8	30
Total for group 2		300
Total for all Voters		1000

Table 9: Initial Votes for multiple seat election

got 700 votes collectively, the distribution amongst candidates is fairly distorted. Similarly group 2 candidates have collectively got 300 votes and for them also distribution is similar to group 1.

Candidate has to get $\left(\frac{1000}{4+1} + 1 = 201\right)$ votes to win. If a candidate gets more than 201 votes, excess votes, called surplus, should be transferred to continuing candidates.

In the Table 9, candidate number 1 has got more than 201 votes resulting in 199 surplus votes. These votes should be transferred to continuing candidates by secondary voting. We invite 400 voters who have voted for candidate 1 to vote again. Details of this voting are given in Table 10.

In all, 210 voters have voted in secondary voting and 199 surplus votes are to be transferred. Each vote in secondary voting has value of $\left(\frac{199}{210}\right)$. The votes are transferred to candidates as listed in Table 11. Total Votes for all candidates are listed in Table 12.

After this round of transfer, no new candidate exceeds winning

Candidate Number	**Number of Votes**
2	0
3	100
4	50
5	25
6	20
7	9
8	6
Total Voted	210
Not Voted	190
Total	**400**

Table 10: First Incremental Votes for multiple seat election

target (known as *Quota*). As a result, we eliminate the candidate with lowest number of votes. In this case, candidate number 8 has the lowest number of votes, so we invite voters who have voted for candidate 8 (original as well as in first incremental voting) to vote again. To minimize number of iterations, let us select bottom 3 (from number of votes point of view) as sequential or combined elimination is not going to impact the result. In actual process, we may have rule based batch selection or we may go for sequential elimination. In the example, we have assumed that batching of candidates to be eliminated does not impact final result.

Candidate 8 (36 Votes), 7 (69 votes) and 4 (72 votes) are selected for casting secondary vote. Continuing candidates are 2, 3, 5 and 6. The voters can cast votes to any of these 4 continuing candidates. They have freedom of not voting. Their ballots have different value, original ballots have value 1 where as ballots received in transfer have value 0.948. This fact should be considered when we transfer votes. Since transfer value cannot be greater than original value, we do not calculate votes on the basis of transferable ballots and votes to be transferred as it would inflate the value of the ballot. Considering this fact, we will transfer votes at the value they are received.

Candidate Number	Number of Ballots	Number of Votes
1	-210	-199
2	0	0
3	100	95
4	50	47
5	25	24
6	20	19
7	9	9
8	6	6
Rounding Adjustment	0	-1
Total	**0**	**0**

Table 11: First Transfer of Votes for multiple seat election

Details of second incremental voting is given in Table 13.

After transferring the votes received in second incremental voting, status is as per Table 14. To accommodate the table in page width, we have abbreviated Candidate to Cand, Ballots to B, Votes to V, Exhausted to Ex and Rounding Adjustment to Rnd. Ballots have different values for transfer. These values are specified in second row.

After second transfer, candidate number 2 and 3 exceed the *Quota*. These two candidates are declared as elected. They have 16 and 25 surplus votes respectively. Usually surplus votes are small numbers for winners who win as result of incremental transfer. If we consider all ballots for transfer of surplus votes, transfer value becomes very small and we may have to invote relatively big number of voters. Since impact of this is not much, we invite only the voters who have voted for new winners in last transfer. So, we invite voters who have voted for candidate number 2 and 3 in last round of incremental voting to cast their votes to continuing candidates. Now continung candidates are candidate number 5 and 6. Number of voters are 17 and 58 for 16 and 25 votes respectively.

To keep the things simple, let us assume that all the voters who

Candidate Number	**Original Ballots**	**Original Votes** Value 1	**First Transfer Ballots**	**First Transfer Votes** Value 0.948	**Total Ballots**	**Total Votes**
1	400	400	-210	-199	190	201
2	200	200	0	0	200	200
3	75	75	100	95	175	170
4	25	25	50	47	75	72
5	120	120	25	24	145	144
6	90	90	20	19	110	109
7	60	60	9	9	69	69
8	30	30	6	6	36	36
Rounding Adjustment	0	0	0	-1	0	-1
Total	**1000**	**1000**	**0**	**0**	**1000**	**1000**

Table 12: Votes after first transfer for multiple seat election

had voted for candidate 2 vote for 5 and others vote for 6. As a result, 16 votes get transferred to candidate number 5 and 25 votes get transferred to candidate number 6. Resultant votes for candidate number 5 and 6 become 191 and 187.

Details of votes for all candidates after the transfer described above is as given in Table 16

The situation here is interesting. No candidate exceeds the *Quota*. Only two candidates are continuing and one of them is to be elected. If we select the one with the lowest number of votes for elimination, the other one gets elected irrespective of the outcome. So, we do not go for additional round of voting and declare candidate with higher number of votes (candidate number 5 in this case) as elected, though the candidate does not exceed the *Quota*.

Candidate number 6 represents all the losers in the result. We can see that all the winners do not exceed the *Quota* but 'the loser' (representative of all the losers put together), has lesser votes compared to all other (winner) candidates. In this case only one candidate has won without reaching *Quota*. There may be more

Ballot Values	From 8		From 7		From 4	
	1	**0.948**	**1**	**0.948**	**1**	**0.948**
To Candidate						
2			5		12	
3	1	2	1	3	11	40
5	9		22			
6	12	3	28	4		6
Not Voted	8	1	4	2	2	4
Total	30	6	60	9	25	50

Table 13: Second Incremental Voting

winners who do not reach the *Quota*. There are several factors which can cause this.

It is not possible to implement this kind of election as it compromises the secrecy of the vote. Moreover, voters cannot be called again (possibly many times) to vote. We can achieve the same result by changing voting process. Rather than voting for a candidate (as suggested in the hypothetical system), voters give their choice in terms of preference number. The most preferred should be given number 1, the next one 2, subsequent $3, 4, \cdots$, and so on. They can give as many preference numbers as they wish and maximum number of preference numbers consumed is independent of number of seats. The only condition is that the preference number 1 must be given, no preference number can be given again and no preference number should be skipped.

Meaning of preference numbers is that initial vote goes to preference number 1. If the candidate wins, surplus votes are transferred at fractional value ($surplus\ votes\ /\ transferrable\ ballots$) to next preferred candidate. This takes care of callig for voting for surplus transfer. If a candidate gets eliminated, ballot gets transferred to the candidate having lowest possible higher preference number of continuing candidate at current value. This takes care of calling voters who have voted to candidate being eliminated.

If this type of voting is used, we get the result the same as we get

	After Last Transfer		From 8				From 7				From 4				Total	
			1		0.948		1		0.948		1		0.948			
Cand	B	V	B	V	B	V	B	V	B	V	B	V	B	V	B	V
1	190	201	0	0	0	0	0	0	0	0	0	0	0	0	190	201
2	200	200	0	0	0	0	5	5	0	0	12	12	0	0	217	217
3	175	170	1	1	2	2	1	1	3	3	11	11	40	38	233	226
4	75	72	0	0	0	0	0	0	0	0	-25	-25	-50	-47	0	0
5	145	144	9	9	0	0	22	22	0	0	0	0	0	0	176	175
6	110	109	12	12	3	3	28	28	4	4	0	0	6	6	163	162
7	69	69		0		0	-60	-60	-9	-9		0		0	0	0
8	36	36	-30	-30	-6	-6	0	0	0	0	0	0	0	0	0	0
Ex	0	0	8	8	1	1	4	4	2	2	2	2	4	4	21	21
Rnd	0	-1												-1		-2
Tot	1000	1000	0	0	0	0	0	0	0	0	0	0	0	0	1000	1000

Table 14: After Second Transfer

in the hypothetical system without compromising secrecy of the vote and calling voters again to vote.

1.3 *Single Transferrable Preferential Voting*

Organizations like State Bar Councils, University Senates [6], Institute of Chartered Accountants of India and Indian Rajyasabha use Single Transferrable Preferentual Voting for election. Exact implementation details vary amongst organizations. Initial value of ballots is 100 for Bar Council where as the same is 1 for others. Another variation could be selection of candidate for elimination in case of equal number of votes. Some organizations use earliest variation where as other use latest variation (the one with lower votes at that stage gets eliminated). One more variation could be number of candidates to be elected under some reservation like duration of organization membership.

1.4 *Conclusion*

In traditional system, for single seat election candidate getting highest number of votes is declared as elected. Similarly, in multi-

	After Last Transfer		From 2 0.941		From 3 0.569		Total	
Cand	B	V	B	V	B	V	B	V
1	190	201	0	0	0	0	190	201
2	217	217	-17	-16	0	0	200	201
3	233	226	0	0	-58	-33	175	201
4	0	0	0	0	0	0	0	0
5	176	175	17	16	0	0	193	191
6	163	162	0	0	58	33	221	187
7	0	0	0	0	0	0	0	0
8	0	0	0	0	0	0	0	0
Ex	21	21	0	0	0	0	21	21
Rnd	0	-2						-2
Tot	1000	1000	0	0	0	0	1000	1000

Table 15: After Third Transfer

seat election, candidates are arranged in descending order of number of votes and as many candidates as there are seats from top are declared as elected. If we analyze the election results, we find that vote share of political parties and number of elected candidates do not match very well. Major reason for this is splitting of amongst losers. We can overcome this by preferential voting and transferring surplus votes of winner (in case of multi-seat election) and all votes of prospective losers (getting lowest number votes). This results in proportional representation in case of multi-seat election and electing candidate getting absolute majority votes in case of single-seat election.

Significant shortcoming of this system is the effort required in counting of votes. If number of candidates are more compared to number of seats then the effort required in counting of votes is significantly more. Similar to electronic voting machine (EVM) based general elections, introduction of electronic voting would result in smoother counting.

Candidate	Votes
1	201
2	201
3	201
4	0
5	191
6	187
7	0
8	0
Exhausted	21
Rounding Error	-2
Total	1000

Table 16: Final Result for Multiple Seat Election

Acknowledgement

The authors thank DST-FIST file # MSI-097 for technical support to the Department of Mathematics.

Suggested Reading

[1] Plulrality Voting *https://en.wikipedia.org/wiki/Plurality_voting*

[2] First-past-the-post voting *https://en.wikipedia.org/wiki/First-past-the-post_voting*

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[5] Chartered Accountants (Election to the Council) Rules, 2006 (Specified by the Central Government) *https://www.icai.org/new_post.html?post_id=272&c_id=244*

[6] GUJARAT UNIVERSITY HAND-BOOK PART-II: Statute 165 *http://www.gujaratuniversity.ac.in/web/data/pdfs/gu-act/Guj_Uni_Handbook_Part-B_Statutes.pdf*