DESIGN OF EQUIRIPPLE FIR FILTER USING REMEZ EXCHANGE ALGORITHM

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***Abstract*—Our project aim is to design FIR Equiripple filters with least possible computational cost and low processing time using Remez algorithm. The computational cost is lowered when compared to windowing methods like Kaiser, hamming and frequency sampling method by minimizing the filter coefficients in the standard filter equation h(n) for desired frequency bands i. e., Low pass, High pass Band pass and Band stop filters respectively. The processing time for determining the coefficients of h(n) is lowered by designing Equiripple filters by remez exchange algorithm. This paper discusses the advantages and applications of using remez algorithm instead direct use of window methods.To get slightly finer solution than approximation problem we use this remez algorithm which in a effect goes a step beyond the minimax approximation algorithm.**

Keywords: Remez algorithm, h(n), FIR, Kaiser, Hamming, Window, Equiripple filters

I. INTRODUCTION

The development in digital signal processing(DSP) is at tremendous rate now-a-days.Digital Signal Processing involves the manipulation of information so that said information can be observed, analyzed, or transformed into a separate form of signal.The field of DSP has always been driven by the advances in DSP applications and in scaled Very-Large-Scale-Integrated(VLSI) Technologies. Digital filter is the key in DSP units which processes the signal that is converted into digital format which is already converted from Analog to Digital by Analog-to-Digital-converter (ADC) in digital circuit.Digital filters have various advantages over Analog filters such as easily programmable, easy to design and implement, lower dependence on time and temperature of working environment, design of digital circuit is compact and cost efficient. These advantages of digital filters makes itmuch more efficient way to design VLSI circuits.

The main two types of filters in Digital Signal Processing are:FIR and IIR filters.FIR filters are mostly used in DSP design due to its two major advantages over IIR. First one is the property of Linear Phase i.e., it ensures the phase is a linear function of the frequency and all frequencies are delayed by the same amount of time, thereby eliminating the possibility of phase distortion which is the reason for processing Audio signals in FIR.The another merit is FIR filters are always stable i.e. for a finite input, the output is always finite.The use of FIR filter has a demerit that it requires large number of filter coefficients which makes the computation complex and overall processing time increases whereas IIR filters can satisfy the constraints with lesser number of coefficients.Our project aims to use FIR filters for DSP with lower number of filter coefficients.

II. METHODOLOGY

There are essentially two well-known methods for FIR filter design namely:

1. The window method

2. Optimal filter design methods

A. Window Method:

The window method is used for designing ideal frequency response of desired filter is equal to 1 for the pass band and 0 for stop band and the filter impulse response is obtained by taking the Discrete Fourier Transform (DFT) of the ideal frequency response. To create a Finite Impulse Response (FIR) filter, the time domain filter coefficients must be restricted in number by multiplying by a window function of a finite width. Basic windowing technique like Rectangular windowing has time domain close to desiredvalue but has side lobes at band edges. Blackman window has better stop-band attenuation and group delay but a wide transition band. To suppress the side lobes and make the filter frequency response approximate more closely to the ideal, the width of the window must be increased and the window function tapered down to zero at the ends. This will increase the width of the transition region between the pass and stop bands which is undesired for an efficient FIR filter.

Window methods are used for designing digital FIR filters easily and it is more preferred due to availability of well defined equations for calculating window coefficient for designing filters.This gives lower design flexibility especially in low pass filter design.Kaiser window is used to get lowest order of the filter to meet given specifications when compared to other windowing techniques. Kaiser window provides the optimal digital filter design due to the presence of a parameters which allows adjustment of the compromise between the overshoot reduction and transition region width spreading for a filter. But while comparing Kaiser window with optimal filter design method, optimal filter design method holds advantageous positiondue to equiripple filter design found to be most suitable and optimized method to meet given specification without overperforming and with lower number of filter coefficients .

Equiripple:

An equiripple filter is normally a filter with ripples of equal height. The magnitude response of actual digital filters may exhibit ripples. For example, the magnitude response of a finite impulse response low pass filter may have ripples close to its cutoff frequency, because the typical filter construction will use continuous functions (e.g., with the Fourier transform) to approximate a discontinuous ideal magnitude response.It is to be noted that the design of equiripple filters is such that the height of these ripples can be controlled. This itself is not unique of equiripple filters.

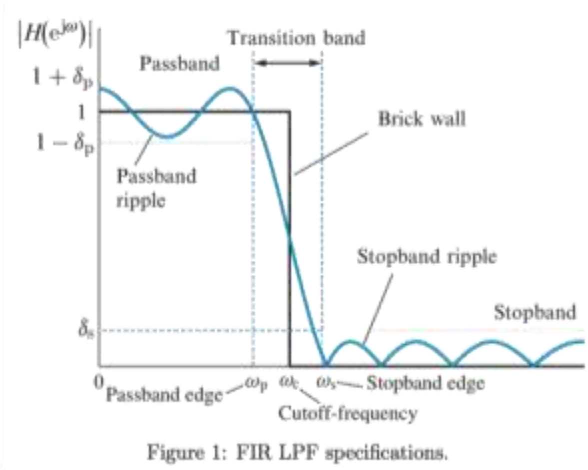

Fig 1: Equiripple and Ideal Low Pass Filter Frequency Response

Design of Optimum equiripple linear-Phase FIR Filters:

Usually we use window method and frequency sampling method ,which are the simple designing techniques for linear-phase FIR filters. But these simple design methods have some minor disadvantages , which becomesunwanted for some applications. And the major issue is they lack the precise control for critical frequencies such as w_p and w_s.

The filter design we use here is Chebyshev approximation. This design is known as an optimum design criterion in the sense that the weightedapproximated error in between the frequency response we desire and the actual frequency response is spread evenly across the passband and evenly across the stopband of the filter by minimizing the maximum error. Theoutputs of this filter design consists of ripples in both the passband and the stopband.

To describe the design procedure, let us consider the design of a lowpass filer with passband edge frequency w_p and the stopband edge frequency w_sFrom the general specifications of the passband, the filter frequency response satisfies the condition.

$$1\text{-}\delta_1 \leq H_r(\omega) \leq 1+\delta_1\,, \quad |\omega| \leq \omega_p$$

Similarly, in the stopband, the filter frequency response is specified to fall between the limits $\pm\delta_2$, that is,

$$-\delta_2 \leq H_r(\omega) \leq \delta_2\,, \quad |\omega| > \omega_s$$

Thus δ1 represents the ripple in the passband and δ_2 represents the attenuation or ripple in the stopband. The remaining filter parameter is M, the filter length or the number of filter coefficients

Let us focus on the four different case that result in a linear-phase FIR filter.

Case	Filter Type	Q(ω)	P(ω)
1.Symmetric unit response	h(n)=h(M-1-n) and M odd	1	$\sum_{k=0}^{(M-1)/2} a(k)\cos\omega k$
2.Symmetric unit sample response.	H(n)=h(M-1-n) and M even	Cos(ω/2)	$\sum_{k=0}^{(M/2)-1} \tilde{b}(k)\cos\omega k$
3.Antisymmetric unit sample response.	H(n)=-h(M-1-n) and M odd	Sin(ω)	$\sum_{k=0}^{(M-3)/2} \tilde{c}(k)\cos\omega k$
4.Antisymmetric unit sample response.	H(n)=-h(M-1-n) and M even	Sin(ω/2)	$\sum_{k=0}^{(M/2)-1} \tilde{d}(k)\cos\omega k$

Table 1 : Real-Valued Frequency Response Functions for Linear -Phase FIR Fiters

The expression for $H_r(\omega)$ in these four cases are summarized in table. we note that the rearrangements that we made in case 2,3 and 4 have allowed us to express $H_r(\omega)$ as

$$H_r(\omega)=Q(\omega)P(\omega)$$

where P(ω) has the common form

$$P(\omega) = \sum_{k=0}^{L} \alpha(k)\cos\omega k$$

With {α(k)} representing the parameters of the filter, which are linearly related to the units sample response h(n) of the FIR filter. The upper limit L in the sum is L=(M-1)/2 for case1, L=(M-3)/2 for case 3, and L=M/2-1 for case and case 4.

In addition to the common framework given above for the representation of $H_r(\omega)$, we also define the real-valued desired frequency response $H_{dr}(\omega)$and the weighting function W(ω) on the approximation error. The real-valued desired frequency response $H_{dr}(\omega)$ is defined to be unity in the passband and zero in the stopband. For example below figure 2.2 illustrates several different type of characteristics for $H_{dr}(\omega)$. the weighting function on the approximation error allows us to choose the relative size of the errors in the different frequency bands(I.e., in the passband and in the stopband. in particular, it is convenient to normalize W(ω) to unity in the stopband and set W(ω)=δ_2/δ_1 in the pssband, that is,

$$W(\omega) = \begin{cases} \delta_2/\delta_1 & passband \\ 1 & stopband \end{cases}$$

Then we simply W(ω) in the passband to reflect our emphasis on the relative size of the ripple in the stopband to the ripple in the passband.

With the specification of $H_{dr}(\omega)$ and W(ω), we can now define the weighted approximation error as

$$E(\omega) = W(\omega)[H_{dr}(\omega) - H_r(\omega)]$$
$$E(\omega) = W(\omega)[H_{dr}(\omega) - Q(\omega)P(\omega)]$$
$$E(\omega) = W(\omega)Q(\omega)\left[H_{dr}(\omega)/Q(\omega) - P(\omega)\right]$$

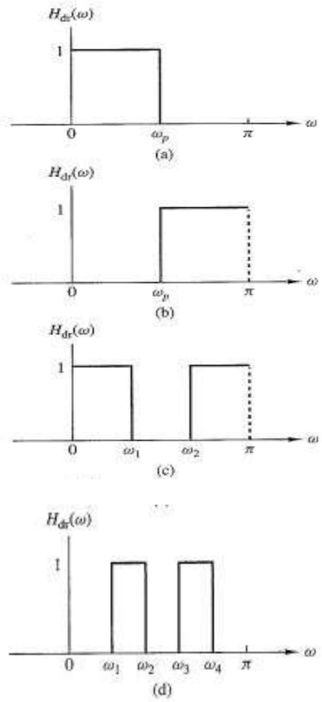

Fig 2:Desired frequency response characteristics for different type offilters

Desired frequency response characteristics for different type of filters. For mathematical convenience, we define a modified weighting function Ŵ(ω) and a modified desired function response $\hat{H}_{dr}(\omega)$ as

$$\hat{W}(\omega)=W(\omega)Q(\omega)$$

$$\hat{H}_{dr}(\omega)=H_{dr}/\,Q(\omega)$$

Then the weighted approximation error may be expressed as

$$E(\omega)=\hat{W}(\omega)[\hat{H}_{dr}(\omega)-P(\omega)]$$

For all four different types of linear-phase FIR filters.

Given the error function E(ω), the chebyshev approximation problem is basically to determine the filter parameters {α(k)} that minimize the maximum absolute value of E(ω) over the frequency bands in which the approximation is to be performed .In mathematical terms, we seek the solutionto the problem

$$\min_{over\{\alpha(k)\}}[\max_{\omega\in S} |E(\omega)|] = \min_{over\{\alpha(k)\}}[\max_{\omega\in S} |\hat{W}(\omega)[H_{dr}(\omega) - \hat{\sum_{k=0}^{L}}\alpha(k)\cos\omega k]|]$$

Where S represents the set (disjoint union) of frequency bands over which the optimization is to be performed .Basically , the set S consists of the passbands and stopbands of the desired filter.

B.REMEZ ALGORITHM:

Remez algorithm is a slight variation of Parks–McClellan Algorithm which is specifically used for FIR filter design. Parks–McClellan Algorithm is the efficient method among Optimal filter design methods: 1. Least square method. 2. Equiripple method. 3. Maximally flat. 4. Generalized equiripple. 5. Constrained band equiripple.

Parks–McClellan Algorithm is an iterative process which starts with an assumption of filter coefficients at the extreme points and calculates the error obtained by taking the coefficients and changes the value based on the error function value which depicts the deviation of obtained response A(ω) from the desired response D(ω) of the filter to be designed.The process of calculating the coefficients is stated by following statements:

1.Assume an initial value for the amplitude at L+2 extremal frequencies.

2.Compute δ using the equations given.

3.Using Lagrange Interpolation, we compute the dense set of samples of A(ω) over the passband and stopband.

4.Determine the new L+2 largest extrema.

5.If the alternation theorem is not satisfied, then we go back to (2) and iterate until the alternation theorem is satisfied.

6.If the obtained response is equal to desired response then the alternation theorem is satisfied, then we compute h(n) which is impulse response of the desired FIR filter.

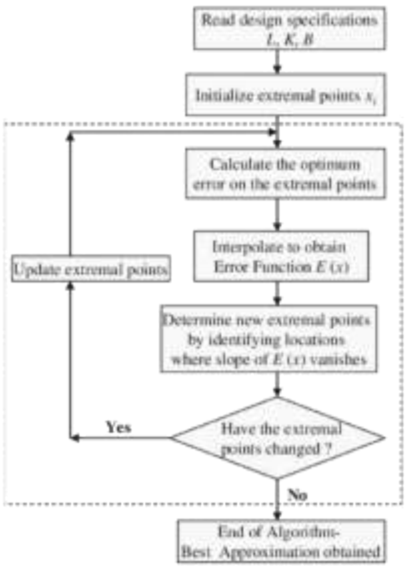

Fig 3: Flowchart depicting the iterative process of Remez Algorithm for finding filter coefficients.

For finding best results for a filter the Error function should be minimized to zero.

$$E(\omega)=W(\omega)(A(\omega)-D(\omega))$$

A(ω): Amplitude Function

D(ω) : desired (real-valued) amplitude function

W(ω) : non-negative weighting function,

Designing the linear-phase filter which minimizes the weighted Chebyshev error, defined by

$$\|E(\omega)\|_\infty = \max_{\omega\in[0,\pi]} |W(\omega)\,(A(\omega) - D(\omega))| \quad \text{--(3)}$$

$$D(\omega)=\begin{cases} 1 & 0<\omega<\omega_0 \quad (passband) \\ 0 & \omega_0<\omega<\pi \quad (stopband) \end{cases} \quad --(4)$$

$$W(\omega)=\begin{cases}K_p & 0\le\omega\le\omega_p\\ 0 & \omega_p<\omega<\omega_s \quad --(5)\\ K_s & \omega_s\le\omega\le\pi\end{cases}$$

$$R := M + 2 \qquad (6)$$

$$E(\omega_i) = c \cdot (-1)^i *\|E(\omega)\|_\infty \text{ for } i = 1, \ldots, R$$

where c is either 1 or −1.

$$\delta = c *\|E(\omega)\|_\infty \quad (7)$$

$$W(\omega_i)(A(\omega_i)-D(\omega_i)) = (-1)^i\delta \cdots > (8)$$
$$A(\omega_i)-D(\omega_i) = (-1)^i\delta / W(\omega_i) \cdots > (9)$$

This procedure is repeated until the error function is minimized to get response near to ideal response of the desired filter.

III. RESULT

Here, remez gives better coefficient results with less number of computations and less computational cost as we can observe from the below results we compared the remez algorithm with kaiser and hamming, where remez have equirriples in passband and stopband and is executed in less time when compared with other windowing techniques and with less noise.

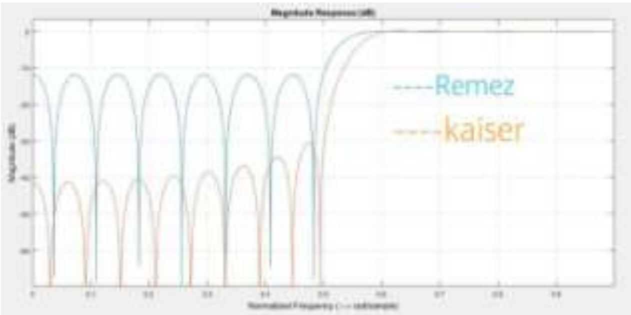

Fig 4: The above figure shows the comparison between magnitude response of remez exchange and kaiser window

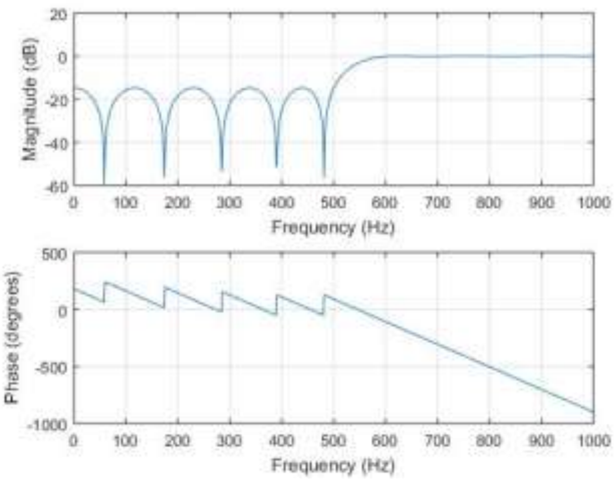

Fig 5: The above figure is the magnitude response and phase response of remez exchange

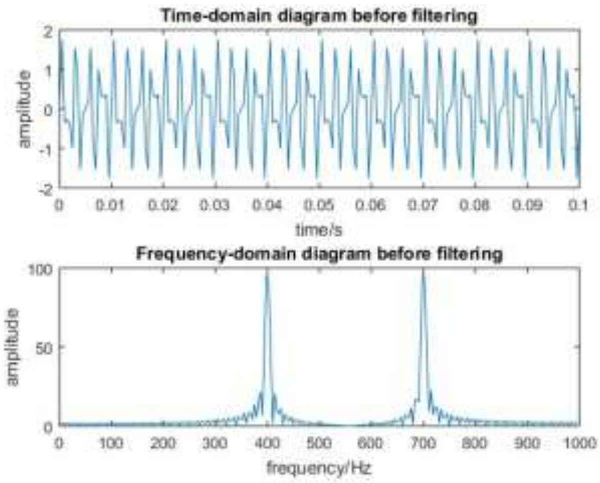

Fig 6: Time domain and frequency domain before filtering for FIR

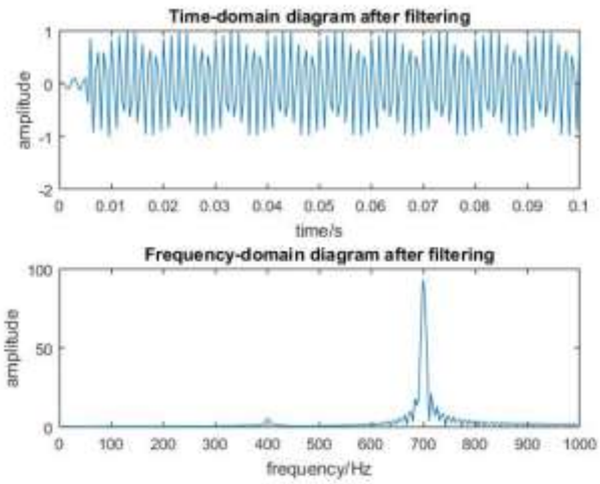

Fig 7: Time domain and Frequency domain after filtering

From Fig. 7 we can observe that the lower frequency component in stopband experience attenuation and there is no frequency component in stopband and no noise after filtering.

The execution time of remez and kaiser window are stated below:

Design Method	**Execution Time**
Remez Exchange	0.5523
Kaiser Window	1.7574

Table 2:Execution time for Remez Exchange algorithm and Kaiser window technique

IV. CONCLUSION

Remez algorithm is one of the Optimal filter design techniques that are used to design best filter for a given length of FIR filter using Equiripple design which is highly advantageous due to its response is near to the ideal frequency response with low order when compared to the window technique. Window technique is simple to design due to existence of well defined formulas in MATLAB toolbox but the disadvantage lies in higher order and more number of coefficients for the same filter design. The equiripple filter has ripples of same gain in stopband where filters designed by window techniques has increasing attenuation in stopband while going away from transition band and there will be distortion at band edge in the pass band.This results in more computations for determining coefficients in window filters which is unrecommended and the approximation error cannot be influenced in different frequency ranges.It is strongly recommended to use minimax strategy for designing a filter which minimizes the error between desired and actual output using a weighted function done by Remez algorithm.

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