A study of balking for the period of vacation in Queuing system

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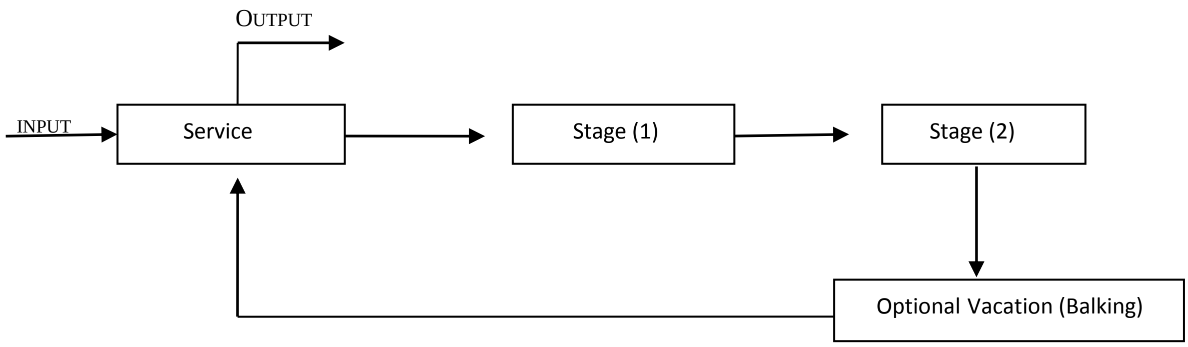
***Abstract*— In this article, we study a M/G/1 Queuing framework with phases of administration and server vacation. Customers show up in groups and the administration is rendered in both the stages consistently to all the showing up customers. The fundamental new suspicion in this paper is investigation of balking. Seeing a long line in the framework, clients may not join the line for administration, they leave the framework without joining the queue. This idea is characterized to be recoiling. We expect phases of administration time and get-away all have a general distribution. The consistent state arrangements have been found by utilizing strengthening variable method. Model is very much legitimized by methods for numerical picture and graphical translation toward the end.**

***Keywords*—** Optional vacation, Balking, Stages of service, Probability generating function, Execution measures.

I. INTRODUCTION

Queuing models with organization obstruction have been analysed by various makers.M/M/1 queue with impatient behaviour of the customer was determined much by Ammar. S.I[1] .Haight.F.A[2] designed a queuing system with balking. Kumar. R and Sharma.S.K[3] investigated the work on reneged customers. Maragathasundari.S et.al [4] prepared a concept of reneging and deferment in the arena of queuing concept. Maragathasundari. S[5] studied an examination on queuing system of general service distribution. Montazer-Haghighi, A.et.al [6] elucidated the multiserver Markovian queuing system with balking and reneging. Maragathasundari.S[7] made an exploration on bulk arrival Queuing model with server vacation. M/G/1 lining framework with expanded excursion and administration interference, various stages in fix process was particularly investigated by Sowmiah , S. also, Maragathasundari, S [8].Non Markovian queue with optional services was all around equipped by Srinivasan, S. and Maragathasundari, S [9]. A Markovian line with shying away, reneging and working get-away was concentrated by Vijaya Lakshmi, P., Goswami, V., and Jyothsna, K., [10].

II. Pictorial description of the Model



III. Algebraic postulation of the Model

Customer's arrival follows a Poisson distribution with arrival rate $\lambda > 0$. The other parameter follows a general distribution.

$P_n^{(1)}(x)$ - Probability that at time t, the server is providing a stage 1 service, there are n customers in the queue excluding the one customer in service and the elapsed service time of this customer is x.

Let $L_1^*(x)$ and $l_1^*(x)$ be the distribution function and the density function of phase service of stage 1. Let $\theta_1(x)$ be the conditional probability of a completion of service of stage 1 and it is given by,

$$\theta_1(x) = \frac{l_1^*(x)}{1-L_1^*(x)} \quad , l_1^*(x) = \theta_1(x)e^{-\int_0^x \theta_1(t)dt} \qquad \text{(i)}$$

Similarly, the process is repeated for service of stage 2 and vacation. Hence we have the following:

$P_n^{(2)}(x)$ - Probability that at time t, the server is providing a stage 2 service, there are n customers in the queue excluding the one customer in service and the elapsed service time of this customer is x.

In case of service of stage 2, we have, $\theta_2(x) = \frac{l_2^*(x)}{1-L_2^*(x)}$ and $l_2^*(x) = \theta_2(x)e^{-\int_0^x \theta_2(t)dt}$ (ii)

$V_n(x)$- denotes the Probability that at time t, the server is on vacation

For the vacation, we have, $\gamma(x) = \frac{l_3^*(x)}{1-L_3^*(x)}$ and $l_3^*(x) = \gamma(x)e^{-\int_0^x \gamma(t)dt}$ (iii)

IV. Prevailing Equations of the model defined

The equations governing the system are as follows:

$$\frac{\partial}{\partial x}P_n^{(1)}(x) + \left(\lambda + \theta_1(x)\right)P_n^{(1)}(x) = \lambda P_{n-1}^{(1)}(x) \qquad (1)$$

$$\frac{\partial}{\partial x}P_0^{(1)}(x) + \left(\lambda + \theta_1(x)\right)P_0^{(1)}(x) = 0 \qquad (2)$$

$$\frac{\partial}{\partial x}P_n^{(2)}(x) + \left(\lambda + \theta_2(x)\right)P_n^{(2)}(x) = \lambda P_{n-1}^{(2)}(x) \qquad (3)$$

$$\frac{\partial}{\partial x}P_0^{(2)}(x) + \left(\lambda + \theta_2(x)\right)P_0^{(2)}(x) = 0 \qquad (4)$$

$$\frac{\partial}{\partial x}V_n(x) + \left(\lambda + \gamma(x)\right)V_n(x) = \lambda b V_{n-1}(x) + \lambda(1-b)V_n(x) \qquad (5)$$

$$\frac{\partial}{\partial x}V_0(x) + \left(\lambda + \gamma(x)\right)V_0(x) = \lambda(1-b)V_0(x) \qquad (6)$$

$$\lambda S = \int_0^\infty P_0^{(2)}(x)\,\theta_2(x)\,dx + \int_0^\infty V_0(x)\gamma(x)\,dx \qquad (7)$$

V. Preliminary and limit Conditions

$$P_n^{(1)}(0) = (1-l)\int_0^\infty P_{n+1}^{(2)}(x)\theta_2(x)\,dx + \int_0^\infty V_{n+1}(x)\gamma(x)\,dx + \lambda S \qquad (8)$$

$$P_n^{(2)}(0) = \int_0^\infty P_{n+1}^{(1)}(x)\theta_1(x)\,dx \qquad (9)$$

$$V_n(0) = l\int_0^\infty P_{n+1}^{(2)}(x)\theta_2(x)\,dx \qquad (10)$$

VI. Supplementary variable performance

We apply the concept of supplementary variable technique for (1) to (10). As a result we get the probability queue size of the system defined.

$$\frac{\partial}{\partial x}P_q^{(1)}(x,z) + \left(\lambda - \lambda z + \theta_1(x)\right)P_q^{(1)}(x,z) = 0 \qquad (11)$$

$$\frac{\partial}{\partial x}P_q^{(2)}(x,z) + \left(\lambda - \lambda z + \theta_2(x)\right)P_q^{(2)}(x,z) = 0 \qquad (12)$$

$$\frac{\partial}{\partial x}V_q(x,z)+\big(b\lambda(1-z)+\gamma(x)\big)V_q(x,z)=0 \qquad (13)$$

$$zP_q^{(1)}(0,z)=(1-l)\int_0^\infty P_q^{(2)}(x,z)\theta_2(x)\,dx+\int_0^\infty V_q(x,z)\gamma(x)\,dx+\lambda S(z-1) \qquad (14)$$

$$P_q^{(2)}(0,z)=\int_0^\infty P_q^{(1)}(x,z)\theta_1(x)\,dx \qquad (15)$$

$$V_q(0,z)=l\int_0^\infty P_q^{(2)}(x,z)\theta_2(x)\,dx \qquad (16)$$

Integrating equation (11) by parts with respect to x, it gives

$$P_q^{(1)}(z)=\left[\frac{1-L_1^*(a)}{a}\right]P_q^{(1)}(0,z) \qquad (17)$$

Where $a=\lambda-\lambda z$ and $L_1^*(a)=\int_0^\infty e^{-(\lambda-\lambda z)x}dL_1(x)$ is the Laplace Stieltjes transform of the service time $L_1(x)$.

Multiplying both side of equation (17) by$\theta_1(x)$, we get

$$\int_0^\infty P_q^{(1)}(x,z)\theta_1(x)\,dx=P_q^{(1)}(0,z)L_1^*(a) \qquad (18)$$

Applying the same procedure for equation (12) to (14), we get

$$P_q^{(2)}(z)=P_q^{(1)}(0,z)L_1^*(a)\left[\frac{1-L_2^*(a)}{a}\right] \qquad (19)$$

$$\int_0^\infty P_q^{(2)}(x,z)\theta_2(x)\,dx=P_q^{(1)}(0,z)L_1^*(a)L_2^*(a) \qquad (20)$$

$$V_q(z)=lP_q^{(1)}(0,z)L_1^*(a)L_2^*(a)\left[\frac{1-L_3^*(c)}{c}\right] \qquad (21)$$

$$\int_0^\infty V_q(x,z)\gamma(x)\,dx=lP_q^{(1)}(0,z)L_1^*(a)L_2^*(a)L_3^*(c) \qquad \text{Where} \quad c=b\lambda(1-z) \qquad (22)$$

Using equation (20) and (22), we get

$$zP_q^{(1)}(0,z)=(1-l)P_q^{(1)}(0,z)L_1^*(a)L_2^*(a)+lP_q^{(1)}(0,z)L_1^*(a)L_2^*(a)L_3^*(c)+\lambda S(z-1) \qquad (23)$$

$$P_q^{(1)}(0,z)=\frac{\lambda S(z-1)}{z-(1-l)L_1^*(a)L_2^*(a)-lL_1^*(a)L_2^*(a)L_3^*(c)} \qquad (24)$$

VII. Likelihood capacity function of the queue size

Let $M_q(z)$ be the Probability generating function of the queue size.

Using equation (17), (19), (21) we get,

$$M_q(z)=P_q^{(1)}(z)+P_q^{(2)}(z)+V_q(z)$$

$$=P_q^{(1)}(0,z)\left\{\left[\frac{1-L_1^*(a)}{a}\right]+L_1^*(a)\left[\frac{1-L_2^*(a)}{a}\right]+lL_1^*(a)L_2^*(a)\left[\frac{1-L_3^*(c)}{c}\right]\right\}$$

$$M_q(z)=\frac{\lambda S(z-1)\left\{\left[\frac{1-L_1^*(a)}{a}\right]+L_1^*(a)\left[\frac{1-L_2^*(a)}{a}\right]+lL_1^*(a)L_2^*(a)\left[\frac{1-L_3^*(c)}{c}\right]\right\}}{z-(1-l)L_1^*(a)L_2^*(a)-lL_1^*(a)L_2^*(a)L_3^*(c)} \qquad (25)$$

VIII. Inactive time and Utilization factor

Using the Normalization Condition,

$M_q(1)+S=1$ We have $M_q(z)$ at z=1 becomes an indeterminate form $\frac{0}{0}$.

Hence we apply L' Hopital's rule to get

$$\lim_{z\to 1} M_q(z) = \frac{N''(1)}{D''(1)}$$

From this, the idle time S can be formed

$$S = \frac{D''(1)}{N''(1)+D''(1)}$$

Also, the utilization factor $\rho = 1 - S$ is determined.

IX. Recital procedures of the queuing system

To find the steady state average queue length, L_q, we adopt the following method

$$L_q = \frac{d}{dz} M_q(z) \text{at} z = 1$$

This attains indeterminate form $\frac{0}{0}$. Consider (25) as $M_q(z) = \frac{N(z)}{D(z)}$

$N(z)$ and $D(z)$ are the numerator and denominator of the R.H.S. of (25)

Apply L'Hopital's rule twice on (25) we obtain

$$L_q = \lim_{z\to 1} \frac{D''(z)N'''(z)\text{-}D'''(z)N''(z)}{2(D''(z))^2} = \frac{D''(1)N'''(1)\text{-}D'''(1)N''(1)}{2(D''(1))^2}$$

Where

$$N(z) = \lambda S(z-1)\left\{\left[\frac{1-L_1^*(a)}{a}\right] + L_1^*(a)\left[\frac{1-L_2^*(a)}{a}\right] + lL_1^*(a)L_2^*(a)\left[\frac{1-L_3^*(c)}{c}\right]\right\}$$

$$D(z) = z - (1-l)L_1^*(a)L_2^*(a) - lL_1^*(a)L_2^*(a)L_3^*(c)$$

$$N''(1) = 2\lambda S\{E(L_1) + E(L_2) + lE(L_3)\}$$

$$N'''(1) = 3\lambda S[\lambda E(L_1^2) + \lambda E(L_2^2) - \lambda lbE(L_3^2) + 2\lambda(1+l)(E(L_1) + E(L_2)]$$

$$D''(1) = -\{-lb\lambda^2 E(L_3)[E(L_1^2) + E(L_1)E(L_2) + E(L_2^2)] + lb\lambda^2 E(L_3)[E(L_1) + E(L_2)] + l(b\lambda)^2 E(L_3^2)\}$$

$$D'''(1) = -\{-b^2\lambda^3 lE(L_3^2)[E(L_1^2) + E(L_2^2) + E(L_1)E(L_2)] - lb\lambda^2 E(L_3)[E(L_2^2)E(L_1) + E(L_1^2)E(L_2)] + \lambda l(b\lambda)^2 E(L_3^2)[E(L_1) + E(L_2)] + lb\lambda^3 E(L_3)[E(L_1^2) + E(L_2^2) + 2E(L_1)E(L_2)] + l\lambda(b\lambda)^2 E(L_3^2)[E(L_1) + E(L_2)]\}$$

Other performance measures of the Queuing system can be found using little's law.

X. Numerical justification

The model is well explained by means of numerical illustration as follows:

$E(L_1) = \frac{1}{\theta_1}$ $E(L_2) = \frac{1}{\theta_2}$ $E(L_3) = \frac{1}{\gamma}$

$E(L_1^2) = \frac{2}{\theta_1^2}$ $E(L_2^2) = \frac{2}{\theta_2^2}$ $E(L_3^2) = \frac{2}{\gamma^2}$

$\lambda = 3.6$ $\theta_1 = 2.5$ $\theta_2 = 3.5$ b=1

$l = 0.6, 0.8, 1, 1.3, 1.5$ $\gamma = 2.6, 2.8, 3.3, 3.5, 3.7$

TABLE 1: EFFECT OF PROBABILITY OF TAKING VACATION ON EXECUTION MEASURES

l	Q	ρ	L_q	L	W_q	W
0.6	0.2799	0.7201	12.1259	12.846	3.3683	3.5683
0.8	0.3235	0.6765	9.8878	10.5643	2.7466	2.9345
1	0.3568	0.6432	8.5448	9.188	2.3736	2.5522
1.3	0.3943	0.6057	7.3052	7.9109	2.0292	2.1975
1.5	0.4136	0.5864	6.7543	7.3407	1.8762	2.0391

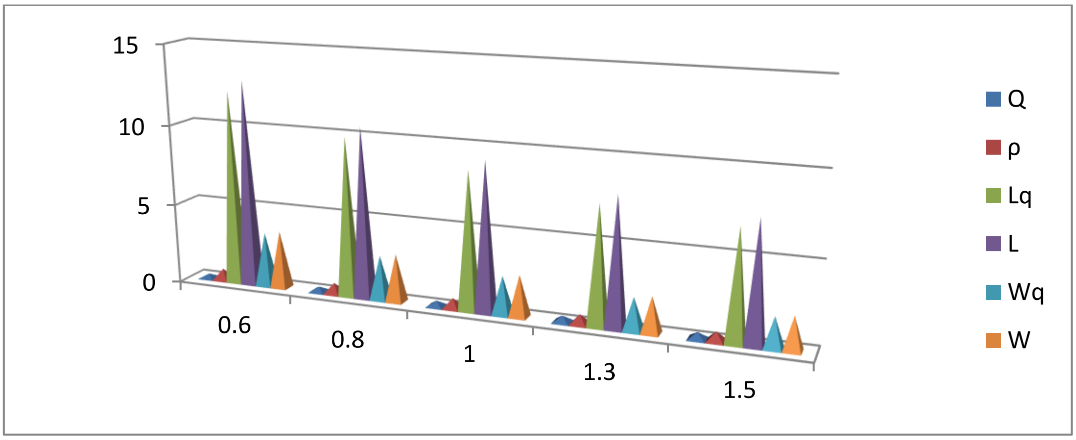
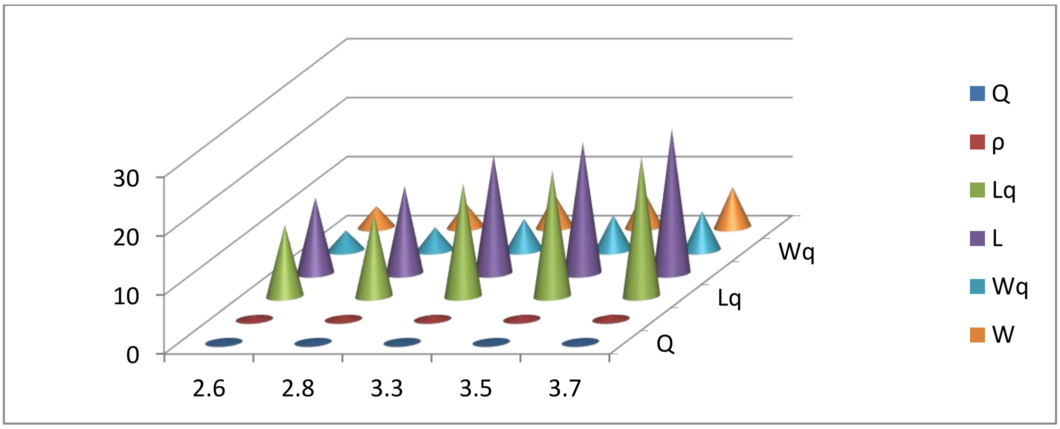


TABLE 2: EFFECT OF PROBABILITY OF COMPLETION OF VACATION TIME ON EXECUTION MEASURES

γ	Q	ρ	L_q	L	W_q	W
2.6	0.2799	0.7201	12.1259	12.846	3.3683	3.5683
2.8	0.2559	0.7441	14.0587	14.8028	3.9052	4.1119
3.3	0.2076	0.7924	19.2484	20.0408	5.3468	5.5669
3.5	0.1919	0.8081	21.4558	22.2639	5.9599	6.1844
3.7	0.1779	0.8221	23.7296	24.5517	6.5916	6.8199



XI. Numerical Analysis Report

TABLE 1 show that increasing the value of l causes decreasing the value of utilization factor, the mean queue size and the mean waiting time for the customers while the server idle time increases. From the TABLE 2, it is clear that, as the probability of completion of vacation increases, it leads to an increase in utilization factor, Lq, L, Wq and W. Also it leads to a decrease in idle time factor. All the results are as expected.

XII. Conclusion

In this article, we talked about a Markovian lining model with phases of administration followed by mandatory server excursion. During the hour of get-away, the idea of recoiling happens. We have inferred the shut structure arrangements of likelihood producing capacity of line size for a non Markovian line with balking happening during server vacation. Model is all around legitimized by methods for numerical delineation and the graphical plots given toward the end shows the impact of get-away on the execution proportions of the lining model.

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