ARITHMETIC OPERATIONS OF SYMMETRIC TRIANGULAR FUZZY RANDOM VARIABLES

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Abstract:
Many methods are available for totalling the arithmetic operations of fuzzy random variables. In this paper, we discuss about the arithmetic operations of the symmetric triangular fuzzy random variables through α – cut values. In this procedure, we get the individual intervals for every α – cut values from the membership function and that the increasing usefulness of symmetric triangular fuzzy random variable in modeling for reliability, economics applications, epidemic models and mathematical tools for proving important results in applied probability. At the end, numerous numerical examples are used to verify the arithmetic operations of symmetric triangular fuzzy random variables.

Keywords:
Fuzzy numbers, Increasing function, Decreasing function, symmetric triangular fuzzy random variable and Arithmetic operations.

I. Introduction

The history of fuzzy set was started from the year 1965, initially it was discovered independently by L.A. Zadeh. [9], it have been useful to handle vague or un cleared boundary concept. The concept of fuzzy random variable was introduced by F´eron [1] in 1976, and developed by some mathematician such as Kwakernaak [2, 3], Puri, Relescu [4, 5], etc., in different views. Fuzzy random variable is an extension of random variable and that the fuzzy random variable is a liquidizer of Randomness and fuzziness. Due to kwakernaak [2] (1978), who viewed a FRV as a vague perception of a crisp, but unobservable random variable. Additionally, Mean and standard deviation of kwakernaak fuzzy random variable are fuzzy. Here, we follow the kwakernaak's fuzzy random variable.

D. Rajan et al., [6] investigate the triangular fuzzy random variable with mean μ and standard deviation σ obtained from the normal distribution and proved some propositions related to various stochastic comparison between two triangular fuzzy random variables.

C. Senthil murugan [7] explore the idea for computing the arithmetic operations between two symmetric trapezoidal fuzzy random variables with mean μ and standard deviation σ. The trapezoidal (a, b, c, d) become triangular fuzzy random variable (a, b, d), when b = c.

V. Vahidi et al., [8] provide a method for determining the coordinate points for the arithmetic operations between two triangular fuzzy numbers. Using these coordinate points, we derive the membership functions of arithmetic operations between two symmetric triangular fuzzy random variables X and Y.

In this paper, we discuss about the triangular fuzzy random variable $(\mu - \sigma, \mu, \mu + \sigma)$ with mean μ and standard deviation σ. Here, the left and right sides of the mean μ are symmetric. For this reason the triangular fuzzy random variable is now called symmetric triangular fuzzy random variable.

The co-ordinate points of symmetric triangular fuzzy random variable are crisp, but the increasing function $(\mu - \sigma, \mu)$ (Def. 2.6) and decreasing function $(\mu, \mu + \sigma)$ (Def. 2.7) under every α cut values are fuzzy. Moreover, the union of increasing and decreasing function constitute the triangle shape and that this shape is convex, so that in triangular fuzzy random variable, every α – cut are convex, every α – cut, α ∈ [0, 1] are also union of increasing decreasing function.

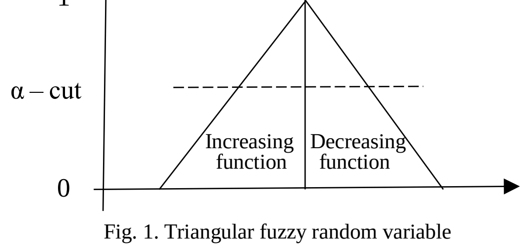

Fig. 1. Triangular fuzzy random variable

The organization of this paper is as follows. In section 2 gives the basic definitions which are useful to build the symmetric triangular fuzzy random variable with mean μ and standard deviation σ respectively. In section 3, we obtain the membership functions of arithmetic operations between two symmetric triangular fuzzy random variables. In section 4, verify the membership function of the arithmetic operations of two symmetric triangular fuzzy random variables through certain numerical examples.

II. Preliminaries:

The following definitions are used in upcoming sections.

A) Definition:

A fuzzy set A is defined as $A = \{X, \mu_A(x): x \in A, \mu_A(x) \in [0, 1]\}$

B) Definition:

The support of fuzzy set A is the set of all points x in X such that $\mu_A(x) > 0$. (i.e.,) support (A) = $\{X / \mu_A(x) > 0\}$.

C) Definition:

The α-cut of α-level set of fuzzy set A is a set consisting of those elements of the universe X whose membership values exceed the threshold level α. (i.e.) $A_\alpha = \{X / \mu_A(x) \geq \alpha\}$

D) Definition:

A fuzzy set A on R must possess at least the following three properties to qualify as a fuzzy number.

i. A must be a normal fuzzy set.
ii. A_α must be closed interval for every $\alpha \in [0, 1]$.
iii. The support of A, ${}^{0+}A$ must be bounded.

Among the various shapes of fuzzy number, triangular fuzzy number is the most popular one.

E) Definition:

The symmetric triangular fuzzy number is a fuzzy number represented with 3 − tuples $A = (a_1, a_2, a_3)$. This representation is interpreted as membership function and holds the following conditions.

i. a_1 to a_2 is increasing function
ii. a_2 to a_3 is decreasing function
iii. a_2 is mean (middle or symmetric point)

$$\mu_A(X) = \begin{cases} 0, & \text{for } X \leq a_1 \\ \dfrac{x - a_1}{a_2 - a_1}, & \text{for } a_1 \leq X \leq a_2 \\ \dfrac{a_3 - x}{a_3 - a_2}, & \text{for } a_2 \leq X \leq a_3 \\ 0, & \text{for } a_3 \geq X \end{cases}$$

Now, the α-cut of the symmetric triangular fuzzy number $[X_\alpha^L, X_\alpha^U]$ is

$$(X_\alpha^L - (a_2 - a_1)\alpha - a_1) \geq 0, \qquad (X_\alpha^U + (a_3 - a_2)\alpha - a_2) \leq 0$$

Applying the values of A (a_1, a_2, a_3) = A $(\mu_1 - \sigma_1, \mu_1, \mu_1 + \sigma_1)$,

We get $X_\alpha^L - (a_2 - a_1)\alpha - a_1 = (X_\alpha^L + (1 - \alpha)\sigma - \mu) \geq 0$, $\quad X_\alpha^U + (a_3 - a_2)\alpha - a_2 = (X_\alpha^U - (1 - \alpha)\sigma - \mu) \leq 0$

Therefore, the α - cut of the symmetric triangular fuzzy random variable is P $\{(X_\alpha^L + (1-\alpha)\sigma - \mu) \geq 0 \vee (X_\alpha^U - (1-\alpha)\sigma - \mu) \leq 0\}$

F) Definition:

The probability distribution function is given by $F(x) = \begin{cases} 0 & x \leq a \\ \frac{(x-a)}{(b-a)} & a < x < b \\ 1 & x \geq b \end{cases}$

Where $f(x) = \frac{1}{(b-a)}$ is the probability density function in $a \leq x \leq b$.

Now, we construct the distribution function of the triangular fuzzy number as follows:

$$(a, b, c) = (\mu_1 - \sigma_1, \mu_1, \mu_1 + \sigma_1) \text{ is } \frac{(x-a)}{(b-a)} \geq \alpha$$

$$\frac{X_\alpha^L - (\mu_1 - \sigma_1)}{\sigma_1} \geq \alpha$$

$$X_\alpha^L - (\mu_1 - \sigma_1) \geq \alpha\sigma_1$$

$\therefore F(X) = a_1^\alpha = P\ \{(X_\alpha^L - (\alpha-1)\sigma_1 - \mu_1) \geq 0\}$ which is the required distribution (increasing) function of the triangular fuzzy random variable.

G) Definition:

The complementary probability distribution or survival function is given by

$$G(x) = \begin{cases} 1 & x \leq b \\ 1 - \frac{(x-b)}{(c-b)} & b < x < c \\ 0 & x \geq c \end{cases}$$

Where $g(x) = \frac{1}{(c-b)}$ is the probability density function in $b \le x \le c$ and $F(x) = 1 - G(x)$.
Now, we construct the survival function of the triangular fuzzy numbers as follows:
$(a, b, c) = (\mu_1 - \sigma_1, \mu_1, \mu_1 + \sigma_1)$

$$1 - \frac{(x-b)}{(c-b)} = 1 - \frac{(X_\alpha^L - \mu_1)}{(\mu_1 + \sigma_1) - \mu_1} \ge \alpha$$
$$\frac{\sigma_1 - X_\alpha^L + \mu_1}{\sigma_1} \ge \alpha$$
$$X_\alpha^U \le -(\alpha - 1)\,\sigma_1 + \mu_1$$

$\therefore$ $G(x) = \overline{F(X)} = a_3^\alpha = P\{(X_\alpha^U + (\alpha - 1)\sigma_1 - \mu_1) \le 0\}$ is the required survival (decreasing) function of the triangular fuzzy random variable.

So that each α - cut is the union of distribution and survival function. Moreover, the triangular fuzzy random variable is union of increasing (distribution) and decreasing (survival) function.

III. Arithmetic operations of symmetric triangular fuzzy random variables

In this section, we discuss about arithmetic operations between two symmetric triangular fuzzy random variables X and Y with mean μ_1, μ_2 and standard deviation σ_1, σ_2 respectively and its corresponding membership functions are
$X = P\{(X_\alpha^L + (1-\alpha)\sigma_1 - \mu_1) \ge 0 \vee (X_\alpha^U - (1-\alpha)\sigma_1 - \mu_1) \le 0\}$, $Y = P\{(Y_\alpha^L + (1-\alpha)\sigma_2 - \mu_2) \ge 0 \vee (Y_\alpha^U - (1-\alpha)\sigma_2 - \mu_2 \le 0\}$.

A. *Addition of symmetric triangular fuzzy random variables*

1) Theorem:

Let X and Y be two symmetric triangular fuzzy random variables with mean μ_1, μ_2 and standard deviation σ_1, σ_2 respectively, then symmetric triangular fuzzy random variable (X+Y) is also a symmetric triangular fuzzy random variable with mean $\mu_1+\mu_2$ and standard deviation $\sigma_1+\sigma_2$ respectively.

Proof:

Mean (X) = μ_1 and mean (Y) = μ_2, then mean (X + Y) = $\mu_1+\mu_2$. According to J. Vahidi and S.Rezvani [8], we get the co – ordinate point of (X+Y) is $\big((\mu_1+\mu_2) - (\sigma_1 + \sigma_2), \mu_1 + \mu_2, (\mu_1 + \mu_2) + (\sigma_1 + \sigma_2)\big)$, its membership function

$$\mu_A(X+Y) = \begin{cases} 0 & , \quad \text{for } (X+Y) \le (\mu_1+\mu_2) - (\sigma_1+\sigma_2) \\ \dfrac{(X+Y)_\alpha^L - (\mu_1+\mu_2) + (\sigma_1+\sigma_2)}{\sigma_1+\sigma_2} & , \quad \text{for } (\mu_1+\mu_2) - (\sigma_1+\sigma_2) \le (X+Y) \le (\mu_1+\mu_2) \\ \dfrac{(\mu_1+\mu_2) + (\sigma_1+\sigma_2) - (X+Y)_\alpha^U}{\sigma_1+\sigma_2} & , \quad \text{for } (\mu_1+\mu_2) \le (X+Y) \le (\mu_1+\mu_2) + (\sigma_1+\sigma_2) \\ 0 & , \quad \text{for } (X+Y) \ge (\mu_1+\mu_2) + (\sigma_1+\sigma_2) \end{cases}$$

Now, we obtain the lower and upper α - cut of symmetric triangular fuzzy random variable (X+Y) as shown below:

Derivation of lower limit

$$\frac{(X+Y)_\alpha^L - (\mu_1 + \mu_2) + (\sigma_1 + \sigma_2)}{\sigma_1+\sigma_2} \ge \alpha$$
$$(X+Y)_\alpha^L - (\mu_1 + \mu_2) \ge (\sigma_1 + \sigma_2)(\alpha - 1)$$
$$((X+Y)_\alpha^L - (\mu_1+\mu_2) + (1-\alpha)(\sigma_1 + \sigma_2)) \ge 0$$
$$P\{(X+Y)_\alpha^L - (\mu_1+\mu_2) + (\sigma_1 + \sigma_2)(1-\alpha) \ge 0\}$$

Derivation of upper limit

$$\frac{(\mu_1 + \mu_2) + (\sigma_1+\sigma_2) - X}{\sigma_1+\sigma_2} \ge \alpha$$
$$(\mu_1+\mu_2) - (X+Y)_\alpha^U \ge (\sigma_1 + \sigma_2)(\alpha - 1)$$
$$((X+Y)_\alpha^U - (\mu_1+\mu_2) - (\sigma_1+\sigma_2)(1-\alpha)) \le 0$$
$$P\{((X+Y)_\alpha^U - (\mu_1+\mu_2) - (\sigma_1 + \sigma_2)(1-\alpha)) \le 0\}$$

Hence, the membership function of sum of symmetric triangular fuzzy random variable (X+Y) is

$$P\{((X+Y)_\alpha^L - (\mu_1 + \mu_2) + (\sigma_1 + \sigma_2)(1-\alpha)) \ge 0 \vee ((X+Y)_\alpha^U - (\mu_1+\mu_2) - (\sigma_1+\sigma_2)(1-\alpha)) \le 0\} \qquad (4.1)$$

This result is also a symmetric triangular fuzzy random variables (X+Y) with mean $\mu_1+\mu_2$ and standard deviation $\sigma_1+\sigma_2$ respectively.

B. *Subtraction of symmetric triangular fuzzy random variables*

2) Theorem:

Let X and Y be two symmetric triangular fuzzy random variables with mean μ_1, μ_2 and standard deviation σ_1, σ_2 respectively, then symmetric triangular fuzzy random variable (X-Y) is again a symmetric triangular fuzzy random variable with mean μ_1-μ_2 and standard deviation $\sigma_1+\sigma_2$ respectively.

Proof:

Mean (X) = μ_1 and mean (Y) = μ_2, then mean (X − Y) = $\mu_1 - \mu_2$. According to J. Vahidi and S.Rezvani [8], we obtain the co – ordinate point of (X−Y) is $\big((\mu_1 - \mu_2) - (\sigma_1+\sigma_2), \mu_1 - \mu_2, (\mu_1 - \mu_2) + (\sigma_1 + \sigma_2)\big)$, its membership function

$$\mu_A(X-Y)=\begin{cases}0 & ,\ \text{for } (X\text{-}Y)\le(\mu_1\text{-}\mu_2)-(\sigma_1+\sigma_2)\\ \dfrac{(X\text{-}Y)^L_\alpha-(\mu_1\text{-}\mu_2)+(\sigma_1+\sigma_2)}{\sigma_1+\sigma_2} & ,\ \text{for } (\mu_1\text{-}\mu_2)-(\sigma_1+\sigma_2)\le(X\text{-}Y)\le(\mu_1\text{-}\mu_2)\\ \dfrac{(\mu_1+\mu_2)+(\sigma_1+\sigma_2)-(X\text{-}Y)^U_\alpha}{\sigma_1+\sigma_2} & ,\ \text{for } (\mu_1\text{-}\mu_2)\le(X\text{-}Y)\le(\mu_1\text{-}\mu_2)+(\sigma_1+\sigma_2)\\ 0 & ,\ \text{for } (X\text{-}Y)\ge(\mu_1\text{-}\mu_2)+(\sigma_1+\sigma_2)\end{cases}$$

Thereafter, we obtain the membership function of (X-Y) is as follows:

Derivation of lower limit

$$\frac{(X\text{-}Y)^L_\alpha-(\mu_1\text{-}\mu_2)+(\sigma_1+\sigma_2)}{\sigma_1+\sigma_2}\ge\alpha$$

$$(X\text{-}Y)^L_\alpha-(\mu_1\text{-}\mu_2)\ge(\sigma_1+\sigma_2)(\alpha\text{-}1)$$

$$((X\text{-}Y)^L_\alpha-(\mu_1\text{-}\mu_2)+(1\text{-}\alpha)(\sigma_1+\sigma_2))\ge 0$$

$$P\{(X\text{-}Y)^L_\alpha-(\mu_1\text{-}\mu_2)+(\sigma_1+\sigma_2)(1\text{-}\alpha)\ge 0\}$$

Derivation of upper limit

$$\frac{(\mu_1-\mu_2)+(\sigma_1+\sigma_2)-(X-Y)^U_\alpha}{\sigma_1+\sigma_2}\ge\alpha$$

$$(\mu_1\text{-}\mu_2)\text{-}(X\text{-}Y)^U_\alpha\ge(\sigma_1+\sigma_2)(\alpha\text{-}1)$$

$$((X\text{-}Y)^U_\alpha-(\mu_1\text{-}\mu_2)-(\sigma_1+\sigma_2)(1\text{-}\alpha))\le 0$$

$$P\{((X\text{-}Y)^U_\alpha-(\mu_1\text{-}\mu_2)-(\sigma_1+\sigma_2)(1\text{-}\alpha))\le 0\}$$

Hence, the membership function of subtraction of symmetric triangular fuzzy random variable (X-Y) is

$$P\{((X-Y)^L_\alpha+(1-\alpha)(\sigma_1+\sigma_2)-(\mu_1-\mu_2))\ge 0\vee((X-Y)^U_\alpha-(1-\alpha)(\sigma_1+\sigma_2)-(\mu_1-\mu_2))\le 0 \qquad (4.2)$$

This result is also a symmetric triangular fuzzy random variables (X –Y) with mean $(\mu_1\text{-}\mu_2)$ and standard deviation $(\sigma_1+\sigma_2)$ respectively.

C. *Multiplication of symmetric triangular fuzzy random variables*

3) Theorem:

Let X and Y be two symmetric triangular fuzzy random variables with mean μ_1,μ_2 and standard deviation σ_1,σ_2 respectively, then multiplication of symmetric triangular fuzzy random variables (X·Y) is a triangular fuzzy random variable with mean $\mu_1\mu_2$.

Proof:

Mean $(X)=\mu_1$ and mean $(Y)=\mu_2$, then mean $(X\cdot Y)=\mu_1\mu_2$. According to Vahidi. J, Rezvani. S [8], we derive the coordinate points of multiplication of two symmetric triangular fuzzy random variables is $((\mu_1-\sigma_1)(\mu_2-\sigma_2),(\mu_1\mu_2),(\mu_1+\sigma_1)(\mu_2+\sigma_2),)$, its membership function is

$$\mu_A(X\cdot Y)=\begin{cases}0, & \text{for } 0\le(X\cdot Y)\le(\mu_1-\sigma_1)(\mu_2-\sigma_2)\\ \dfrac{(X\cdot Y)-(\mu_1-\sigma_1)(\mu_2-\sigma_2)}{(\mu_1\mu_2)-(\mu_1-\sigma_1)(\mu_2+\sigma_2)}, & \text{for } (\mu_1-\sigma_1)(\mu_2-\sigma_2)\le(X\cdot Y)\le(\mu_1\mu_2)\\ \dfrac{(\mu_1+\sigma_1)(\mu_2+\sigma_2)-(X\cdot Y)}{(\mu_1+\sigma_1)(\mu_2+\sigma_2)-(\mu_1\mu_2)}, & \text{for } (\mu_1\mu_2)\le(X\cdot Y)\le(\mu_1+\sigma_1)(\mu_2+\sigma_2)\\ 0, & \text{for } (X\cdot Y)\ge(\mu_1+\sigma_1)(\mu_2+\sigma_2)\end{cases}$$

First, we obtain the lower α - cut of (X·Y) is given below:

$$\frac{(X\cdot Y)^L_\alpha-(\mu_1-\sigma_1)(\mu_2-\sigma_2)}{(\mu_1\mu_2)-(\mu_2-\sigma_2)(\mu_1-\sigma_1)}\ge\alpha$$

On simplification, we get

$$(X\cdot Y)^L_\alpha-(\alpha-1)(\mu_1\sigma_2+\mu_2\sigma_1-\sigma_1\sigma_2)-\mu_1\mu_2\ge 0$$

$$P\{((X\cdot Y)^L_\alpha-(\alpha-1)(\mu_1\sigma_2+\mu_2\sigma_1-\sigma_1\sigma_2)-\mu_1\mu_2)\ge 0\}$$

Which is the required lower α – cut of the multiplication of symmetric triangular fuzzy random variables.

Secondly, we derive the upper α- cut of (X·Y) is as follows:

$$\frac{(\mu_1+\sigma_1)(\mu_2+\sigma_2)-(X\cdot Y)^U_\alpha}{(\mu_1+\sigma_1)(\mu_2+\sigma_2)-(\mu_1\mu_2)}\ge\alpha$$

$$(X\cdot Y)^U_\alpha+(\alpha-1)(\mu_1\sigma_2+\mu_2\sigma_1+\sigma_1\sigma_2)-\mu_1\mu_2\ge 0$$

$$P\{((X\cdot Y)^U_\alpha+(\alpha-1)(\mu_1\sigma_2+\mu_2\sigma_1+\sigma_1\sigma_2)-\mu_1\mu_2)\ge 0\}$$

Which is the required upper α- cut of multiplication of symmetric triangular fuzzy random variables.

Hence, the membership function of multiplication of two symmetric triangular fuzzy random variables (X·Y) is

$$P\{((X\cdot Y)^L_\alpha-(\alpha-1)(\mu_1\sigma_2+\mu_2\sigma_1-\sigma_1\sigma_2)-\mu_1\mu_2)\ge 0\vee(X\cdot Y)^U_\alpha+(\alpha-1)(\mu_1\sigma_2+\mu_2\sigma_1+\sigma_1\sigma_2)-\mu_1\mu_2)\ge 0 \qquad (4.3)$$

This result is a triangular fuzzy random variable, but not symmetric.

D. ***Division of symmetric triangular fuzzy random variables***

4) Theorem:

Let X and Y be two symmetric triangular fuzzy random variables with mean μ_1, μ_2 and standard deviation σ_1, σ_2 respectively, then division of symmetric triangular fuzzy random variables (X/Y) is a triangular fuzzy random variable with mean μ_1/μ_2, but not symmetric.

Proof:

Mean $(X) = \mu_1$ and mean $(Y) = \mu_2$, then mean $(X/Y) = \mu_1/\mu_2$. According to Vahidi. J, Rezvani. S [8], we get the coordinate point of division of two symmetric triangular fuzzy random variables is $(X/Y) = \left(\frac{\mu_1-\sigma_1}{\mu_2+\sigma_2}, \frac{\mu_1}{\mu_2}, \frac{\mu_1+\sigma_1}{\mu_2-\sigma_2}\right)$, and its membership function is

$$\mu(X/Y) = \begin{cases} 0, & \text{for } 0 \le (X/Y) \le \frac{\mu_1-\sigma_1}{\mu_2+\sigma_2} \\ \frac{(X/Y) - \left(\frac{\mu_1-\sigma_1}{\mu_2+\sigma_2}\right)}{\frac{\mu_1}{\mu_2} - \left(\frac{\mu_1-\sigma_1}{\mu_2+\sigma_2}\right)}, & \text{for } \frac{\mu_1-\sigma_1}{\mu_2+\sigma_2} \le (X/Y) \le \frac{\mu_1}{\mu_2} \\ \frac{\left(\frac{\mu_1+\sigma_1}{\mu_2-\sigma_2}\right) - (X/Y)}{\left(\frac{\mu_1+\sigma_1}{\mu_2-\sigma_2}\right) - \frac{\mu_1}{\mu_2}}, & \text{for } \frac{\mu_1}{\mu_2} \le (X/Y) \le \frac{\mu_1+\sigma_1}{\mu_2-\sigma_2} \\ 0, & \text{for } (X/Y) \ge \frac{\mu_1+\sigma_1}{\mu_2-\sigma_2} \end{cases}$$

First, we derive the lower α - cut of (X/Y) is as follows

$$\frac{(X/Y) - \frac{\mu_1-\sigma_1}{\mu_2+\sigma_2}}{\frac{\mu_1}{\mu_2} - \left(\frac{\mu_1-\sigma_1}{\mu_2+\sigma_2}\right)} \ge \alpha$$

On simplification we get,

$$(X/Y) \ge \frac{\alpha(\mu_1\sigma_2 + \mu_2\sigma_1)}{\mu_2(\mu_2+\sigma_2)} + \left(\frac{\mu_1-\sigma_1}{\mu_2+\sigma_2}\right)$$

$$P\{((X/Y)_\alpha^L - \frac{\alpha(\mu_1\sigma_2 + \mu_2\sigma_1)}{\mu_2(\mu_2+\sigma_2)} - \left(\frac{\mu_1-\sigma_1}{\mu_2+\sigma_2}\right) \ge 0\}$$

Which is the required lower α - cut of the triangular fuzzy random variables.

Secondly, we obtain the upper α −cut of (X/Y) is as follows

$$\frac{\frac{\mu_1+\sigma_1}{\mu_2-\sigma_2} - (X/Y)}{\left(\frac{\mu_1+\sigma_1}{\mu_2-\sigma_2}\right) - \frac{\mu_1}{\mu_2}} \ge \alpha$$

On simplification, we get

$$(X/Y) \le -\frac{\alpha(\mu_1\sigma_2 + \mu_2\sigma_1)}{\mu_2(\mu_2-\sigma_2)} + \left(\frac{\mu_1+\sigma_1}{\mu_2-\sigma_2}\right)$$

$$P\{((X/Y)_\alpha^U + \frac{\alpha(\mu_1\sigma_2 + \mu_2\sigma_1)}{\mu_2(\mu_2-\sigma_2)} - \left(\frac{\mu_1+\sigma_1}{\mu_2-\sigma_2}\right) \le 0\}$$

Which is the required upper α- cut of triangular fuzzy random variable.

Hence, the membership function of division of two symmetric triangular fuzzy random variables (X/Y) is

$$P\{((X/Y)_\alpha^L - \frac{\alpha(\mu_1\sigma_2 + \mu_2\sigma_1)}{\mu_2(\mu_2+\sigma_2)} - \left(\frac{\mu_1-\sigma_1}{\mu_2+\sigma_2}\right) \ge 0 \vee ((X/Y)_\alpha^U + \frac{\alpha(\mu_1\sigma_2 + \mu_2\sigma_1)}{\mu_2(\mu_2-\sigma_2)} - \left(\frac{\mu_1+\sigma_1}{\mu_2-\sigma_2}\right)) \le 0\} \quad (4.4)$$

This result is a triangular fuzzy random variable, but not symmetric.

IV. Numerical examples

A. Example:

Let X and Y be two symmetric triangular fuzzy random variables with mean $\mu_1 = 3$, $\mu_2 = 6$ and standard deviation $\sigma_1 = 1$, $\sigma_2 = 0.5$ respectively, then find symmetric triangular fuzzy random variable (X+Y) with mean 9 and standard deviation 1.5 respectively.

Solution:

The membership function of (X+Y) is

$$P\{((X+Y)_\alpha^L - (\mu_1+\mu_2)+(\sigma_1+\sigma_2)(1-\alpha)) \geq 0 \vee ((X+Y)_\alpha^U - (\mu_1+\mu_2) - (\sigma_1+\sigma_2)(1-\alpha)) \leq 0\}$$

Substitute $\mu_1 = 3$, $\mu_2 = 6$ and $\sigma_1 = 1$, $\sigma_2 = 0.5$, we get

$$P\{((X+Y)_\alpha^L - 9+(1-\alpha)1.5) \geq 0 \vee ((X+Y)_\alpha^U - 9 - (1-\alpha)1.5) \leq 0\}$$

Put $\alpha = 1$, $P\{((X+Y)_\alpha^L - 9) \geq 0 \vee ((X+Y)_\alpha^U - 9) \leq 0\}$

Put $\alpha = 0$, $P\{((X+Y)_\alpha^L - 7.5) \geq 0 \vee ((X+Y)_\alpha^U - 10.5) \leq 0\}$

Therefore, the sum (X+Y) of symmetric triangular fuzzy random variable is (7.5, 9, 10.5) with mean $\mu_1 + \mu_2 = 9$ and standard deviation $\sigma_1 + \sigma_2 = 1.5$.

B. Example:

The following table shows that the remaining arithmetic operations such as subtraction, multiplication and division between two symmetric triangular fuzzy random variables (X and Y) by using various parameter values, solved in the above similar manner.

X		Y		Operations / Equation	$\mu = 0$ (X_α^L, X_α^U)	$\mu = 1$ (X_α^L, X_α^U)	Co - ordinate point	Mean, Standard deviation
μ_1	σ_1	μ_2	σ_2					
2	1	3	1	Subtraction / (4.2)	(-3, 1)	-1	(-3, -1, 1)	$\mu_1 - \mu_2 = -1$, $\sigma_1 + \sigma_2 = 2$
4	.5	4	1	Multiplication/ (4.3)	(10.5, 21.5)	16	(10.5, 16, 21.5)	$\mu_1 \cdot \mu_2 = 16$ Asymmetric
3	1	5	2	Division / (4.4)	$\frac{2}{7}, \frac{4}{3}$	$\frac{3}{5}$	$\frac{2}{7}, \frac{3}{5}, \frac{4}{3}$	$\frac{\mu_1}{\mu_2} = \frac{3}{5}$ Asymmetric

Table 5.1 Arithmetic operations of (X and Y) by using various means and standard deviations.

V. Conclusion

In this paper, we have to compute the membership functions of addition, subtraction, multiplication and division of symmetric triangular fuzzy random variables and it has been tested using a numerical example. The results show that the membership function described in this paper perform well. The arithmetic operations are useful in further development of triangular fuzzy random variable.

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