ON α-Δ-OPEN SETS AND GENERALIZED Δ-CLOSED SETS IN TOPOLOGICAL SPACES

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ABSTRACT. The aim of this paper, we introduced and studied α-Δ-open sets in topological spaces. We offer a new class of sets called $g\Delta$-closed sets in topological spaces and we study some of its basic properties. we introduce $g\Delta$-interior and $g\Delta$-closure and study some of its basic properties. We introduce $g\Delta$-continuous maps and $g\Delta$-irresolute maps. we introduce the classes of Δ-lc-set, $g\Delta$-lc-set, $g\Delta$-lc*-sets, $g\Delta$-lc**-sets and study some of its basic properties. Finally we introduced and studied ΔLC-continuous, $G\Delta$LC-continuous map and $G\Delta$LC-irresolute map.

1. Introduction and Preliminaries

M. Veera Kumar [18] introduced and studied Δ-open set in topology. T. M. Nour and Ahmad Mustafa Jaber [16] introduced and studied semi-Δ-open sets. S. Ganesan [10] introduced and studied pre-Δ-open sets, b-Δ-open sets and β-Δ-open sets in topological spaces. N. Levine [13] introduced generalised closed sets in topological sapces. S. Ganesan [7] introduced and studied the new operator of open sets and generalized closed sets in topological spaces. In this way, we introduced and studied α-Δ-open sets in topological spaces. We offer a new class of sets called $g\Delta$-closed sets in topological spaces and we study some of its basic properties. we introduce $g\Delta$-interior and $g\Delta$-closure and study some of its basic properties. We introduce $g\Delta$-continuous maps and $g\Delta$-irresolute maps.

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we introduce the classes of Δ-lc-set, $g\Delta$-lc-set, $g\Delta$-lc*-sets, $g\Delta$-lc**-sets and study some of its basic properties. Finally we introduced and studied ΔLC-continuous, $G\Delta$LC-continuous map and $G\Delta$LC-irresolute map.

Definition 1.1. *A subset A of a space (X, τ) is called:*

(1) *α-open set* [15] *if $A \subseteq int(cl(int(A)))$.*

(2) *semi-open set* [12] *if $A \subseteq cl(int(A))$.*

(3) *preopen set* [14] *if $A \subseteq int(cl(A))$.*

(4) *b-open set* [3] *if $A \subseteq int(cl(A)) \cup cl(int(A))$.*

(5) *β-open set* [1] *(= semi-preopen set* [2]*) if $A \subseteq cl(int(cl(A)))$.*

The complements of the above mentioned open sets are called their respective closed sets.

Definition 1.2. *A subset A of a space (X, τ) is called*

(1) *Δ-open* [16, 18] *if $A = (B - C) \cup (C - B)$, where B and C are open subsets of X.*

(2) *semi-Δ-open* [16] *if $A = (B - C) \cup (C - B)$, where B and C are semi-open subsets of X.*

(3) *pre-Δ-open* [10] *if $A = (B - C) \cup (C - B)$, where B and C are pre-open subsets of X.*

(4) *b-Δ-open* [10] *if $A = (B - C) \cup (C - B)$, where B and C are b-open subsets of X.*

(5) *β-Δ-open* [10] *if $A = (B - C) \cup (C - B)$, where B and C are β-open subsets of X.*

The complements of the above mentioned open sets are called their respective closed sets.

Definition 1.3. [18] *Let A be a subset of a space (X, τ), then the Δ-closure of A, denoted by $\Delta cl(A)$ is defined as the intersection of all Δ-closed subsets of X containing A.*

Remark 1.1. *Every α-closed sets is semi-closed sets but not conversely.[See Remark 3.24* [17]*]*

2. α-Δ-open sets

Definition 2.1. *A subset S of a space (X, τ) said to be α-Δ-open set if $S = (A - B) \cup (B - A)$, where A and B are α-open subsets in X.*

The complement of α-Δ-open sets is called α-Δ-closed sets.

We denote the power set of X by P(X).

It is evident that every Δ-open set as also every α-open set is α-Δ-open. But the converse implications are not true in general. Following is an example.

Example 2.1. *Let $X = \{a, b, c\}$ and $\tau = \{\phi, \{a\}, X\}$. Then α-open sets are ϕ, $\{a\}$, $\{a, b\}$, $\{a, c\}$, X ; Δ-open sets are ϕ, $\{a\}$, $\{b, c\}$, X and α-Δ-open sets are power set of X. It is clear that $\{a, b\}$ is α-Δ-open set but it is not Δ-open set. (ii) It is clear that $\{b, c\}$ is α-Δ-open set but it is not α-open set.*

Definition 2.2. *Every α-Δ-open set is semi-Δ-open set but not conversely.*

Proof. Let S be α-Δ-open set. Since every α-open set set is semi-open set. We have S = (A − B) ∪ (B − A), where A and B are semi-open subsets of X. Hence, S is semi-Δ-open set. □

Example 2.2. *Let $X = \{a, b, c\}$ and $\tau = \{\phi, \{a\}, \{b\}, \{a, b\}, X\}$. Then α-Δ-open sets are ϕ, $\{a\}$, $\{b\}$, $\{c\}$, $\{a, b\}$, $\{a, c\}$, X ; semi-Δ-open sets are power set of X. Here $\{b, c\}$ is semi-Δ-open set but it is not α-Δ-open set.*

Remark 2.1. *(1) The union of any two α-Δ-open sets is not a α-Δ-open set.*
(2) The intersection of any two α-Δ-closed sets is not a α-Δ-closed set.

Example 2.3. *Let X and τ as in the Example 2.2. (i) Here $M = \{a\}$ and $N = \{c\}$ are α-Δ-open sets, but $M \cup N = \{a, c\}$ is not a α-Δ-open set. (ii) Here $O = \{a, b\}$ and $P = \{b, c\}$ are α-Δ-closed sets, but $O \cap P = \{b\}$ is not a α-Δ-closed set.*

Definition 2.3. *Let (X, τ) be a topological space and let $A \subseteq X$. Then the union of all α-Δ-open sets contained in A, denoted by α-Δ-int(A), is called the α-Δ-interior of A.*

Theorem 2.1. *Let (X, τ) be a topological space and $A \subset X$. Then, A is α-Δ-open if and only if $A = \alpha$-Δ-int(A).*

Proof. Let A be a α-Δ-open set. Then, A ⊆ A and this implies that A ∈ {U | U is α-Δ-open and U ⊂ A}. Since union of this collection is in A. Therefore, A = α-Δ-int(A).
Conversely, suppose that A = α-Δ-int(A). Hence, A is α-Δ-open. □

Definition 2.4. *Let A be a subset of a space (X, τ) then the α-Δ-closure of A, denoted by α-$\Delta cl(A)$ is defined as the intersection of all α-Δ-closed subsets of X containing A.*

Theorem 2.2. *Let (X, τ) be a topological space and A, $B \subseteq X$. Then, the following statements hold:*

(1) $A \subseteq \alpha$-$\Delta cl(A)$.

(2) α-$\Delta cl(A)$ *is the smallest* α-Δ-*closed set containing* A, *that is* α-$\Delta cl(A) = \bigcap \{ W \mid W$ *is* α-Δ-*closed ans* $A \subseteq W\}$.

(3) A *is* α-Δ-*closed if and only if* $A = \alpha$-$\Delta cl(A)$.

(4) *If* $A \subseteq B$, *then* α-$\Delta cl(A) \subseteq \alpha$-$\Delta cl(B)$.

(5) α-$\Delta cl(A) \cup \alpha$-$\Delta cl(B) \subseteq \alpha$-$\Delta cl(A \cup B)$.

(6) α-$\Delta cl(A \cap B) \subseteq \alpha$-$\Delta cl(A) \cap \alpha$-$\Delta cl(B)$.

Proof. (1) Let x $\in$ A and suppose that x $\notin$ α-Δcl(A). Then, there exits α-Δ-open set V containing x such that V $\cap$ A $= \emptyset$ and this is a contradiction. Therefore, x $\in$ α-Δcl(A).
(2) Let x $\in$ α-Δcl(A). Then, V $\cap$ A $\neq \emptyset$ for every α-Δ-open set V containing x. Now, suppose the contrary, that x $\notin \bigcap \{$ W $\mid$ W is α-Δ-closed ans A $\subseteq$ W$\}$. Then, x $\notin$ W for some α-Δ-closed set W, so x $\in$ X $-$ W for some α-Δ-open set X $-$ W. So, (X$-$ W) $\cap$ A $=$ $\emptyset$ for some α-Δ-open set X $-$ W containing x and this is a contradiction. Therefore, x $\notin$ $\bigcap \{$ W $\mid$ W is α-Δ-closed ans A $\subseteq$ W$\}$. Conversely, let y $\in$ x $\notin \bigcap \{$ W $\mid$ W is α-Δ-closed ans A $\subseteq$ W$\}$. Then, y $\in$W for all α-Δ-closed set W containing A. Now, suppose that y $\notin$ α-Δcl(A). Then, there exits α-Δ-open set V containing y such that V $\cap$ A $= \emptyset$. Therefore, X $-$ V is α-Δ-closed set containing A and y $\notin$ X $-$ V and this is a contradiction. Therefore, y $\in$ α-Δcl(A).
The proof of (3) and (4) are followed directly from the Definition 2.4. (5) and (6) are followed by applying part (4) of this Theorem. □

Theorem 2.3. *Let (X, τ) be a topological space and A, $B \subseteq X$. Then, the following statements hold:*

(1) *If* $A \subseteq B$, *then* α-Δ-$int(A) \subseteq \alpha$-Δ-$int(B)$.

(2) α-Δ-*int(A)* $\cup$ α-Δ-*int(B)* $\subseteq$ α-Δ-*int(A* $\cup B)$.

(3) α-Δ-*int(A* $\cap B)$ $\subseteq$ α-Δ-*int(A)* $\cap$ α-Δ-*int(B)*.

Proof. The proof of (1) is followed directly from the Definition 2.3. (2) and (3) are followed by applying part (1) of this Theorem. □

Theorem 2.4. *Let (X, τ) be a topological space and $A \subseteq X$. Then, the following statements hold:*

(1) α-Δ-*int*$(X \setminus A) = X \setminus \alpha$-$\Delta cl(A)$.

(2) α-$\Delta cl(X \setminus A) = X \setminus \alpha$-$\Delta$-*int(A)*.

(3) $X \setminus \alpha$-$\Delta cl(X \setminus A) = \alpha$-$\Delta$-*int(A)*.

(4) $X \setminus \alpha$-Δ-*int*$(X \setminus A) = \alpha$-$\Delta cl(A)$.

(5) $x \in \alpha$-Δ-*int(A) if and only if there exists a* α-Δ-*open set M such that* $x \in M \subseteq A$.

Proof. Omitted. □

Theorem 2.5. *Let A be a subset of a topological space (X, τ). Then, $x \in \alpha$-$\Delta cl(A)$ if and only if for every α-Δ-open subset M of X containing x, $A \cap M \neq \emptyset$.*

Proof. Let x $\in$ α-Δcl(A) and suppose that M $\cap$ A = $\emptyset$ for some α-Δ-open set M which contains x. Then, (X $\setminus$ M) is α-Δ-closed and A $\subset$ (X $\setminus$ M), thus α-Δcl(A) $\subset$ (X $\setminus$ M). But this implies that x $\in$ (X $\setminus$ M), a contradiction. Thus, A $\cap$ M $\neq \emptyset$.

Conversely, let A $\subseteq$ X and x $\in$ X such that for each α-Δ-open set M_1 which contains x, M_1 $\cap \neq \emptyset$. If x $\notin$ α-Δcl(A), there is a α-Δ-closed set F such that A $\subseteq$ F and x $\notin$ F. Then, (X $\setminus$ F) is a α-Δ-open set with x $\in$ (X $\setminus$ F), and thus (X $\setminus$ F) $\cap$ A $\neq \emptyset$, which is a contradiction. □

Theorem 2.6. *Let (X, τ) be a topological space $A \subseteq X$. Then A is α-Δ-open if and only if for each $s \in A$, there exists a α-Δ-open set D such that $s \in D \subseteq A$.*

Proof. It is obvious. □

Definition 2.5. *A map $f : (X, \tau) \to (Y, \sigma)$ is said to be α-Δ-continuous if $f^{-1}(A)$ is α-Δ-open set in (X, τ) for every Δ-open set A of (Y, σ).*

Theorem 2.7. *A map $f : X \to Y$ is α-Δ-continuous if and only if the inverse image of every Δ-open set in Y is α-Δ-open in X.*

Proof. Let f be α-Δ-continuous and K be any Δ-open set in Y. If $f^{-1}(K) = \emptyset$, then $f^{-1}(K)$ is a α-Δ-open set in X but if $f^{-1}(K) \neq \emptyset$, then there exists $x \in f^{-1}(K)$ which implies $f(x) \in K$. Since f is α-Δ-continuous, then there exists a α-Δ-open set L in X containing x such that $f(L) \subseteq K$. This implies that $x \in L \subseteq f^{-1}(K)$ and hence $f^{-1}(K)$ is α-Δ-open.
Conversely, let K be any Δ-open set in Y containing f(x), then $x \in f^{-1}(K)$ and by hypothesis $f^{-1}(K)$ is a α-Δ-open set in X containing x, so $f(f^{-1}(K)) \subseteq K$. Thus, f is α-Δ-continuous. □

Theorem 2.8. *For a map $f : X \to Y$, the following statements are equivalent:*

(1) *f is α-Δ-continuous.*
(2) *$f^{-1}(K)$ is a α-Δ-open set in X, for each Δ-open subset K of Y.*
(3) *$f^{-1}(F)$ is a α-Δ-closed set in X, for each Δ-closed subset F of Y.*
(4) *$f(\alpha\text{-}\Delta cl(A)) \subseteq \Delta cl(f(A))$, for each subset A of X.*
(5) *$\alpha\text{-}\Delta cl(f^{-1}(B)) \subseteq f^{-1}(\Delta cl(B))$, for each subset B of Y.*
(6) *$f^{-1}(\Delta int(B)) \subseteq \alpha\text{-}\Delta int(f^{-1}(B))$, for each subset B of Y.*

Proof. (1) ⇒ (2): Directly from Theorem 2.7.
(2) ⇒ (3): Let F be any Δ-closed subset of Y. Then, $Y \setminus F$ is a Δ-open subset of Y. By (2), $f^{-1}(Y \setminus F) = X \setminus f^{-1}(F)$ is a α-Δ-open set in X and hence $f^{-1}(F)$ is a α-Δ-closed set in X.
(3) ⇒ (4): Let A be any subset of X. Then, $f(A) \subseteq \Delta cl(f(A))$ and $\Delta cl(f(A))$ is a Δ-closed set in Y. Hence, $A \subseteq f^{-1}(\Delta cl(f(A)))$. By (3), we have $f^{-1}(\Delta cl(f(A)))$ is a α-Δ-closed set in X. Therefore, $\alpha\text{-}\Delta cl(A) \subseteq f^{-1}(\Delta cl(f(A)))$. Hence, $f(\alpha\text{-}\Delta cl(A)) \subseteq \Delta cl(f(A))$.
(4) ⇒ (5): Let B be any subset of Y. Then, $f^{-1}(B)$ is a subset of X. By (4), we have $f(\alpha\text{-}\Delta cl(f^{-1}(B))) \subseteq \Delta cl(f(f^{-1}(B))) \subseteq \Delta cl(B)$. Hence, $\alpha\text{-}\Delta cl(f^{-1}(B)) \subseteq f^{-1}(\Delta cl(B))$.

(5) $\Leftrightarrow$ (6): Let B be any subset of Y. Then, apply (5) to Y $\setminus$ B we obtain α-Δcl(f^{-1}(Y $\setminus$ B))$\subseteq$ f^{-1}(Δcl(Y $\setminus$ B)) $\Leftrightarrow$ α-Δcl(X $\setminus$ f^{-1}(B))$\subseteq$ f^{-1}(Y $\setminus$ Δint(B)) $\Leftrightarrow$ X $\setminus$ α-Δint(f^{-1}(B))$\subseteq$ X $\setminus$ f^{-1}(Δint(B)) $\Leftrightarrow$ f^{-1}(Δint(B)) $\subseteq$ α-Δint(f^{-1}(B)). Thus, f^{-1}(Δint(B))) $\subseteq$ α-Δint(f^{-1}(B)).
(6) $\Rightarrow$ (1): Let x $\in$ X and K be any Δ-open subset of Y containing f(x). By (6), we have f^{-1}(Δint(K)) $\subseteq$ α-Δint(f^{-1}(K)) implies that f^{-1}(K) $\subseteq$ α-Δint(f^{-1}(K)). Hence, f^{-1}(K) is a α-Δ-open set in X which contains x and clearly f(f^{-1}(K)) $\subseteq$ K. Thus, f is α-Δ-continuous. □

Definition 2.6. *A map $f : (X, \tau) \to (Y, \sigma)$ is said to be α-Δ-irresolute if $f^{-1}(V)$ is α-Δ-open set in (X, τ) for every α-Δ-open set V of (Y, σ).*

Theorem 2.9. *Let $f : (X, \tau) \to (Y, \sigma)$ be a map, then the following statements are equivalent:*

(1) *f is α-Δ-irresolute.*
(2) *f(α-Δcl(A)) $\subseteq$ α-Δcl(f(A)) holds for every subset A of X.*
(3) *f^{-1}(B) is α-Δ-closed set in X, for every α-Δ-closed subset B of Y.*

Proof. (2) $\Rightarrow$ (3): Let B be a α-Δ-closed set in Y, then α-Δcl(B) = B. By using (2), we have f(α-Δclf^{-1}(B))$\subseteq$ α-Δcl(B) = B. Thus, (α-Δclf^{-1}(B))$\subseteq$ f^{-1}(B) and hence f^{-1}(B) is α-Δ-closed in X.
(3) $\Rightarrow$ (2): If A $\subseteq$ X, then α-Δcl(f(A)) is α-Δ-closed in Y and by (3) f^{-1}(α-Δcl(f(A))) is α-Δ-closed in X. Furthermore, A $\subseteq$ f^{-1}(f(A)) $\subseteq$ f^{-1}(α-Δcl(f(A))). Thus, α-Δcl(A) $\subseteq$ f^{-1}(α-Δcl(f(A))), consequently, f(α-Δcl(A)) $\subseteq$ f(f^{-1}(α-Δcl(f(A)))) $\subseteq$ α-Δcl(f(A)).
(3) $\Leftrightarrow$ (1): Obvious. □

3. $g\Delta$-closed and $g\Delta$-open sets

Definition 3.1. *A subset A of a space (X, τ) is called a generalized Δ-closed (briefly, $g\Delta$-closed) set if $\Delta cl(A) \subseteq T$ whenever $A \subseteq T$ and T is Δ-open in (X, τ).*
The complement of $g\Delta$-closed set is called $g\Delta$-open set.

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Proposition 3.1. *Every* Δ*-closed set is* $g\Delta$*-closed.*

Proof. Let A be a Δ-closed set and T be any Δ-open set containing A. Since A is Δ-closed, we have Δcl(A) = A $\subseteq$ T. Hence A is $g\Delta$-closed. □

The converse of Proposition 3.1 need not be true as seen from the following example.

Example 3.1. *Let* $X = \{a, b, c, d\}$ *and* $\tau = \{\phi, \{a, b\}, X\}$. *Then* Δ*-closed sets are* ϕ, $\{c, d\}$, $\{a, b, d\}$, $X\}$ *and* $g\Delta$*-closed sets are* ϕ, $\{d\}$, $\{a, c\}$, $\{a, d\}$, $\{b, c\}$, $\{c, d\}$, $\{a, b, c\}$, $\{a, b, d\}$, $\{a, c, d\}$, $\{b, c, d\}$, X. *Here,* $H = \{a, b, c\}$ *is* $g\Delta$*-closed set but it is not* Δ*-closed.*

Proposition 3.2. *If* P *and* Q *are* $g\Delta$*-closed sets, then* $P \cup Q$ *is also a* $g\Delta$*-closed set.*

Proof. Let P and Q are $g\Delta$-closed sets. Then Δcl(P) $\subseteq$ T where P $\subseteq$ T and T is Δ-open and Δcl(Q) $\subseteq$ T where Q $\subseteq$ T and T is Δ-open. Since P and Q are subsets of T, (P $\cup$ Q) is a subset of T and T is Δ-open. Then Δcl(P $\cup$ Q) = Δcl(P) $\cup$ Δcl(Q) which implies that (P $\cup$ Q) is $g\Delta$-closed. □

Remark 3.1. *If* K *and* L *are* $g\Delta$*-closed sets, then* $K \cap L$ *is a not* $g\Delta$*-closed set.*

Example 3.2. *Let* X *and* τ *as in the Example 3.1. Here,* $k = \{a, c\}$ *and* $L = \{b, c\}$ *are* $g\Delta$*-closed sets but* $K \cap L = \{c\}$ *is a not* $g\Delta$*-closed set.*

Proposition 3.3. *If A subset A of* (X, τ) *is a* $g\Delta$*-closed if and only if* $\Delta cl(A) - A$ *does not contain any nonempty* Δ*-closed set.*

Proof. Necessity. Suppose that A is $g\Delta$-closed. Let S be a Δ-closed subset of Δcl(A) $-$ A. Then A $\subseteq$ S^c . Since A is $g\Delta$-closed, we have Δcl(A) $\subseteq$ S^c . Consequently, S $\subseteq$ $(\Delta\text{cl(A)})^c$. Hence, S $\subseteq$ Δcl(A) $\cap$ $(\Delta\text{cl(A)})^c$ $= \phi$. Therefore S is empty.

Sufficiency. Suppose that Δcl(A) $-$ A contains no nonempty Δ-closed set. Let A $\subseteq$ G and G be Δ-closed If Δcl(A) $\neq$ G, then Δcl(A) $\subseteq$ G^c $\neq \phi$. Since Δcl(A) is a Δ-closed set and G^c is a Δ-closed set, Δcl(A) $\cap$ G^c is a nonempty Δ-closed subset of Δcl(A) $-$ A. This is a contradiction. Therefore, Δcl(A) $\subseteq$ G and hence A is $g\Delta$-closed. □

Proposition 3.4. *If A is $g\Delta$-closed in (X, τ) such that A $\subseteq$ B $\subseteq$ Δcl(A), then B is also a $g\Delta$-closed set of (X, τ).*

Proof. Let W be a Δ-open set of (X, τ) such that B $\subseteq$ W. Then A $\subseteq$ W. Since A is $g\Delta$-closed, we get, Δcl(A) $\subseteq$ W. Now Δcl(B) $\subseteq$ Δcl(Δcl(A)) = Δcl(A) $\subseteq$ W. Therefore, B is also a $g\Delta$-closed set of (X, τ). □

Definition 3.2. *The intersection of all Δ-open subsets of (X, τ) containing A is called the Δ-kernel of A and denoted by Δ-ker(A).*

Lemma 3.1. *A subset A of (X, τ) is $g\Delta$-closed if and only if Δcl(A) $\subseteq$ Δ-ker(A).*

Proof. Suppose that A is $g\Delta$-closed. Then Δcl(A) $\subseteq$ T whenever A $\subseteq$ T and T is Δ-open. Let x $\in$ Δcl(A). If x $\notin$ Δ-ker(A), then there is a Δ-open set T containing A such that x $\notin$ T. Since T is a Δ-open set containing A, we have x $\notin$ Δcl(A) and this is a contradiction. Conversely, let Δcl(A) $\subseteq$ Δ-ker(A). If T is any Δ-open set containing A, then Δcl(A) $\subseteq$ Δ-ker(A) $\subseteq$ T. Therefore, A is $g\Delta$-closed. □

Definition 3.3. *A subset A of a space X is said to be $g\Delta$-open if A^C is $g\Delta$-closed.*

The class of all $g\Delta$-open subsets of X is denoted by $g\Delta$o(τ).

Proposition 3.5. *Every Δ-open set is $g\Delta$-open set but not conversely.*

Proof. Omitted. □

Proposition 3.6. *A subset A of a topological space X is said to $g\Delta$-open if and only if P $\subseteq$ Δint(A) whenever A $\supseteq$ P and P is Δ-closed in X.*

Proof. Suppose that A is $g\Delta$-open in X and A $\supseteq$ P, where P is Δ-closed in X. Then A^c $\subseteq$ P^c , where P^c is Δ-open in X. Hence we get Δcl (A^c) $\subseteq$ P^c implies (Δint(A))^c $\subseteq$ P^c . Thus, we have Δint(A) $\supseteq$ P.

conversely, suppose that A^c $\subseteq$ T and T is Δ-open in X then A $\supseteq$ T^c and T^c is Δ-closed then by hypothesis Δint(A) $\supseteq$ T^c implies (Δint(A))^c $\subseteq$ T. Hence Δcl (A^c) $\subseteq$ T gives A^c is $g\Delta$-closed. □

Proposition 3.7. *In a topological space X, for each $x \in X$, either $\{x\}$ is Δ-closed or $g\Delta$-open in X.*

Proof. Suppose that {x} is not Δ-closed in X. Then $\{x\}^c$ is not Δ-open and the only Δ-open set containing $\{x\}^C$ is the space X itself. Therefore, Δcl $(\{x\}^C) \subseteq$ X and so $\{x\}^C$ is $g\Delta$-closed gives {x} is $g\Delta$-open. □

We introduce the following definition.

Definition 3.4. *A space (X, τ) is called a $T\Delta 1/2$-space if every $g\Delta$-closed set is Δ-closed.*

Example 3.3. *Let X and τ as in the Example 2.2. Then $g\Delta$-closed sets are ϕ, $\{a\}$, $\{c\}$, $\{a, b\}$, $\{a, c\}$, $\{b, c\}$, X and Δ-closed sets are ϕ, $\{a\}$, $\{c\}$, $\{a, b\}$, $\{a, c\}$, $\{b, c\}$, X. Thus (X, τ) is a $T\Delta 1/2$-space.*

Example 3.4. *Let X and τ as in the Example 2.1. Then Δ-closed sets are ϕ, $\{a\}$, $\{b, c\}$, X and $g\Delta$-closed sets are power set of X. Thus (X, τ) is not a $T\Delta 1/2$-space.*

Theorem 3.1. *For a space (X, τ) the following properties are equivalent:*

(i) *(X, τ) is a $T\Delta 1/2$-space.*

(ii) *Every singleton subset of (X, τ) is either Δ-closed or Δ-open.*

Proof. (i) → (ii). Assume that for some x ∈ X, the set {x} is not a Δ-closed in (X, τ). Then the only Δ-open set containing $\{x\}^c$ is X and so $\{x\}^c$ is $g\Delta$-closed in (X, τ). By assumption $\{x\}^c$ is Δ-closed in (X, τ) or equivalently {x} is Δ-open.

(ii) → (i). Let A be a $g\Delta$-closed subset of (X, τ) and let x ∈ Δcl(A). By assumption {x} is either Δ-closed or Δ-open. □

Case (a) Suppose that {x} is Δ-closed. If x ∉ A, then Δcl(A) − A contains a nonempty Δ-closed set {x}, which is a contradiction to Proposition 3.3. Therefore x ∈ A.

Case (b) Suppose that {x} is Δ-open. Since x ∈ Δcl(A), {x} ∩ A ≠ ϕ and so x ∈ A. Thus in both case, x ∈ A and therefore Δcl(A) ⊆ A or equivalently A is a Δ-closed set of (X, τ). □

4. $g\Delta$-interior and $g\Delta$-closure

Definition 4.1. *For any $A \subseteq X$, $g\Delta$-int(A) is defined as the union of all $g\Delta$-open sets contained in A. i.e., $g\Delta\text{-}int(A) = \cup\{G : G \subseteq A$ and G is $g\Delta\text{-}open\}$.*

Lemma 4.1. *For any $A \subseteq X$, $int(A) \subseteq g\Delta\text{-}int(A) \subseteq A$.*

Proof. The proof follows from Proposition 3.5. □

Proposition 4.1. *For any $A \subseteq X$, the following holds.*

(1) *$g\Delta$-int(A) is the largest $g\Delta$-open set contained in A.*
(2) *A is $g\Delta$-open if and only if $g\Delta\text{-}int(A) = A$.*

Proposition 4.2. *For any subsets A and B of (X, τ), the following holds.*

(1) $g\Delta\text{-}int(A \cap B) = g\Delta\text{-}int(A) \cap g\Delta\text{-}int(B)$.
(2) $g\Delta\text{-}int(A \cup B) \supseteq g\Delta\text{-}int(A) \cup g\Delta\text{-}int(B)$.
(3) *If $A \subseteq B$, then $g\Delta\text{-}int(A) \subseteq g\Delta\text{-}int(B)$.*
(4) $g\Delta\text{-}int(X) = X$ *and* $g\Delta\text{-}int(\phi) = \phi$.

Definition 4.2. *For every set $A \subseteq X$, we define the $g\Delta$-closure of A to be the intersection of all $g\Delta$-closed sets containing A. i.e., $g\Delta\text{-}cl(A) = \cap\{F : A \subseteq F \in g\Delta\text{-}closed)$.*

Lemma 4.2. *For any $A \subseteq X$, $A \subseteq g\Delta\text{-}cl(A) \subseteq \Delta cl(A)$.*

Proof. The proof follows from Proposition 3.1. □

Remark 4.1. *Both containment relations in Lemma 4.2 may be proper as seen from the following example.*

Example 4.1. *Let X and τ as in the Example 3.4. Let $A = \{a, b\}$. Here $g\Delta\text{-}cl(\{a, b\}) = \{a, b\}$ and so $A \subseteq g\Delta\text{-}cl(A) \subseteq \Delta cl(A)$.*

Proposition 4.3. *For any $A \subseteq X$, the following holds.*

(1) *$g\Delta$-clo(A) is the smallest $g\Delta$-closed set containing A.*

(2) *A is $g\Delta$-closed if and only if $g\Delta$-cl(A) = A.*

Proposition 4.4. *For any two subsets A and B of (X, τ), the following holds.*

(1) *If $A \subseteq B$, then $g\Delta$-cl(A) $\subseteq$ $g\Delta$-cl(B).*

(2) *$g\Delta$-cl($A \cap B$) $\subseteq$ $g\Delta$-cl(A) $\cap$ $g\Delta$-cl(B).*

Proposition 4.5. *Let A be a subset of a space X, then the following are true.*

(1) *$(g\Delta\text{-}int(A))^c = g\Delta\text{-}cl(A^c)$.*

(2) *$g\Delta\text{-}int(A) = (g\Delta\text{-}cl(A^c))^c$.*

(3) *$g\Delta\text{-}cl(A) = (g\Delta\text{-}int(A^c))^c$.*

Proof. (1) Clearly follows from definitions.

(2) Follows by taking complements in (1).

(3) Follows by replacing A by A^c in (1). □

5. $g\Delta$-Continuous maps and Irresolute maps

We introduce the following definition.

Definition 5.1. *A map $f : (X, \tau) \to (Y, \sigma)$ is called Δ-continuous (resp. $g\Delta$-continuous) if $f^{-1}(W)$ is a Δ-closed (resp. $g\Delta$-closed) set of (X, τ) for every Δ-closed set W of (Y, σ).*

Proposition 5.1. *Every Δ-continuous is $g\Delta$-continuous but not conversely.*

Proof. The proof follows from Proposition 3.1. □

Theorem 5.1. *If $f : (X, \tau) \to (Y, \sigma)$ is $g\Delta$-continuous and $g : (Y, \sigma) \to (Z, \gamma)$ is Δ-continuous then $g \circ f : (X, \tau) \to (Z, \gamma)$ is $g\Delta$-continuous.*

Proof. Let G be Δ-closed set in Z. Since g is Δ-continuous, g^{-1}(G) is Δ-closed in Y. Since f is $g\Delta$-continuous, $(g \circ f)^{-1}(G) = f^{-1}(g^{-1}(G))$ is $g\Delta$-closed in X. Therefore g ∘ f is $g\Delta$-continuous. □

Proposition 5.2. *A map f : (X, τ) $\rightarrow$ (Y, σ) is $g\Delta$-continuous if and only if f^{-1}(W) is $g\Delta$-open in (X, τ) for every Δ-open set W in (Y, σ).*

Proof. Let f : (X, τ) $\rightarrow$ (Y, σ) be $g\Delta$-continuous and W be an Δ-open set in (Y, σ). Then W^c is Δ-closed in (Y, σ) and since f is $g\Delta$-continuous, $f^{-1}(W^c)$ is $g\Delta$-closed in (X, τ). But $f^{-1}(W^c) = f^{-1}((W))^c$ and so f^{-1}(W) is $g\Delta$-open in (X, τ).
Conversely, assume that f^{-1}(W) is $g\Delta$-open in (X, τ) for each Δ-open set W in (Y, σ). Let F be a Δ-closed set in (Y, σ). Then F^c is Δ-open in (Y, σ) and by assumption, $f^{-1}(F^c)$ is $g\Delta$-open in (X, τ). Since $f^{-1}(F^c) = f^{-1}((F))^c$, we have f^{-1}(F) is Δ-closed in (X, τ) and so f is $g\Delta$-continuous.

Theorem 5.2. *Let f : (X, τ) $\rightarrow$ (Y, σ) be an $g\Delta$-continuous map. If (X, τ), the domain of f is an $T\Delta 1/2$-space, then f is Δ-continuous.*

Proof. Let W be a Δ-closed set of (Y, σ). Then f^{-1}(W) is a $g\Delta$-closed set of (X, τ), since f is $g\Delta$-continuous. Since (X, τ) is an TΔ1/2-space, then f^{-1}(W) is a Δ-closed set of (X, τ). Therefore f is Δ-continuous. □

Definition 5.2. *Let (X, τ) be a topological space. Let x be a point of X and G be a subset of X. Then G is called an $g\Delta$-neighbourhood of x (briefly, $g\Delta$-nbhd of x) in X if there exists an $g\Delta$-open set S of x such that $x \in S \subseteq G$.*

Proposition 5.3. *Let A be a subset of (X, τ). Then $x \in g\Delta$-cl(A) if and only if for any $g\Delta$-nbhd G_x of x in (X, τ), $A \cap G_x \neq \phi$.*

Proof. Necessity. Assume x $\in g\Delta$-cl(M). Suppose that there is an $g\Delta$-nbhd G of the point x in (X, τ) such that G $\cap$ A = ϕ. Since G is $g\Delta$-nbhd of x in (X, τ), by Definition 5.2, there exists an $g\Delta$-open set S_x such that x $\in S_x \subseteq$ G. Therefore, we have $S_x \cap$ A = ϕ and so A $\subseteq (S_x)^c$. Since $(S_x)^c$ is an $g\Delta$-closed set containing A, we have by Definition 4.2, $g\Delta$-cl(A) $\subseteq (S_x)^c$ and therefore x $\notin g\Delta$-cl(A), which is a contradiction.
Sufficiency. Assume for each $g\Delta$-nbhd G_x of x in (X, τ), A $\cap G_x \neq \phi$. Suppose that x $\notin$ $g\Delta$-cl(A). Then by Definition 4.2, there exists an $g\Delta$-closed set F of (X, τ) such that A $\subseteq$

F and x $\notin$ F. Thus x $\in$ F^c and F^c is $g\Delta$-open in (X, τ) and hence F^c is a $g\Delta$-nbhd of x in (X, τ). But A $\cap$ F^c $=\phi$, which is a contradiction. □

In the next theorem we explore certain characterizations of $g\Delta$-continuous maps.

Theorem 5.3. *Let f : (X, τ) $\rightarrow$ (Y, σ) be a map from a topological space (X, τ) into a topological space (Y, σ) . Then the following statements are equivalent.*

(1) *The map f is $g\Delta$-continuous.*

(2) *The inverse of each Δ-open set is $g\Delta$-open.*

(3) *For each point x in (X, τ) and each Δ-open set C in (Y, σ) with f(x) $\in$ C, there is an $g\Delta$-open set B in (X, τ) such that x $\in$ B, f(B) $\subseteq$ C.*

(4) *The inverse of each Δ-closed set is $g\Delta$-closed.*

(5) *For each x in (X, τ), the inverse of every neighbourhood of f(x) is an $g\Delta$-nbhd of x.*

(6) *For each x in (X, τ) and each neighbourhood N of f(x), there is an $g\Delta$-nbhd G of x such that f(G) $\subseteq$ N.*

(7) *For each subset A of (X, τ), f($g\Delta$-cl(A)) $\subseteq$ Δcl(f(A)).*

(8) *For each subset B of (Y, σ) , $g\Delta$-cl(f^{-1}(B)) $\subseteq$ f^{-1}(Δcl(B)).*

Proof. (1) $\Leftrightarrow$ (2). This follows from Proposition 5.2.

(1) $\Leftrightarrow$ (3). Suppose that (3) holds and let C be an Δ-open set in (Y, σ) and let x $\in$ f^{-1}(C). Then f(x) $\in$ C and thus there exists an $g\Delta$-open set B_x such that x $\in$ B_x and f(B_x) $\subseteq$ C. Now, x $\in$ B_x $\subseteq$ f^{-1}(C) and f^{-1}(C) = $\cup_{x \in f^{-1}(C)} B_x$. Then arbitrary union of $g\Delta$-open set is $g\Delta$-open, f^{-1}(C) is $g\Delta$-open in (X, τ) and therefore f is $g\Delta$-continuous.

Conversely, Suppose that (1) holds and let f(x) $\in$ C. Then x $\in$ f^{-1}(C) $\in$ $g\Delta$o(τ), since f is $g\Delta$-continuous. Let B = f^{-1}(C). Then k $\in$ B and f(B) $\subseteq$ C.

(2) $\Leftrightarrow$ (4). This result follows from the fact if A is a subset of (Y, σ), then $f^{-1}(A^c)$ = $(f^{-1}(A))^c$.

(2) $\Leftrightarrow$ (5). For x in (X, τ), let N be a neighbourhood of f(x). Then there exists an Δ-open set C in (Y, σ) such that f(x) $\in$ C $\subseteq$ N. Consequently, f^{-1}(C) is an $g\Delta$-open set in (X, τ) and x $\in$ f^{-1}(C) $\subseteq$ f^{-1}(N). Thus f^{-1}(N) is an $g\Delta$-nbhd of x.

(5) ⇔ (6). Let x ∈ X and let N be a neighbourhood of f(x). Then by assumption, G = f^{-1}(N) is an $g\Delta$-nbhd of x and f(G) = f(f^{-1}(N)) ⊆ N.

(6) ⇔ (3). For x in (X, τ), let C be an Δ-open set containing f(x). Then C is a neighborhood of f(x). So by assumption, there exists an $g\Delta$-nbhd G of x such that f(G) ⊆ C. Hence there exists an $g\Delta$-open set B in (X, τ) such that x ∈ B ⊆ G and so f(B) ⊆ f(G) ⊆ C.

(7)⇔ (4). Suppose that (4) holds and let A be a subset of (X, τ). Since A ⊆ f^{-1}(A), we have A ⊆ f^{-1}(Δcl(f(A))). Since Δcl(f(A)) is a Δ-closed set in (Y, σ) , by assumption f^{-1}(Δcl(f(A))) is an $g\Delta$-closed set containing A. Consequently, $g\Delta$-cl(A) ⊆ f^{-1}(Δcl(f(A))). Thus f($g\Delta$-cl(A)) ⊆ f(f^{-1}(Δcl(f(A)))) ⊆ Δcl(f(A)).

Conversely, suppose that (7) holds for any subset A of (X, τ). Let F be a Δ-closed subset of (Y, σ) . Then by assumption, f($g\Delta$-cl(f^{-1}(F))) ⊆ Δcl(f(f^{-1}(F))) ⊆ Δcl(F) = F. i.e., $g\Delta$-cl(f^{-1}(F))⊆ f^{-1}(F) and so f^{-1}(F) is $g\Delta$-closed.

(7) ⇔ (8). Suppose that (7) holds and B be any subset of (Y, σ) . Then replacing A by f^{-1}(B) in (7), we obtain f($g\Delta$-cl(f^{-1}(B))) ⊆ Δcl(f(f^{-1}(B))) ⊆ Δcl(B). i.e., $g\Delta$-cl(f^{-1}(B)) ⊆ f^{-1} Δcl(B).

Conversely, suppose that (8) holds. Let B = f(A) where A is a subset of (X, τ). Then we have, $g\Delta$-cl(A) ⊆ $g\Delta$-cl(f^{-1}(B)) ⊆ f^{-1}(Δcl(f(A)) and so f($g\Delta$-cl(A)) ⊆ Δcl(f(A)). □

Definition 5.3. *A map f : (X, τ) → (Y, σ) is called $g\Delta$-irresolute if f^{-1}(W) is a $g\Delta$-closed set of (X, τ) for every $g\Delta$-closed set W of (Y, σ).*

Theorem 5.4. *Every $g\Delta$-irresolute map is $g\Delta$-continuous but not conversely.*

Proof. Let f : (X, τ) → (Y, σ) be a $g\Delta$-irresolute map. Let W be a Δ-closed set of (Y, σ). Then by the Proposition 3.1, W is $g\Delta$-closed. Since f is $g\Delta$-irresolute, then f^{-1}(W) is a $g\Delta$-closed set of (X, τ). Therefore f is $g\Delta$-continuous. □

Theorem 5.5. *Let f : (X, τ) → (Y, σ) and g : (Y, σ) → (Z, γ) be any two maps. Then*

(1) *$g \circ f$ is $g\Delta$-continuous if g is Δ-continuous and f is $g\Delta$-continuous.*
(2) *$g \circ f$ is $g\Delta$-irresolute if both f and g are $g\Delta$-irresolute.*
(3) *$g \circ f$ is $g\Delta$-continuous if g is $g\Delta$-continuous and f is $g\Delta$-irresolute.*

Proof. Omitted. □

Definition 5.4. *A map f : (X, τ) $\rightarrow$ (Y, σ) is called*

(1) *contra Δ-continuous if f^{-1}(W) is a Δ-closed set of (X, τ) for every Δ-open set W of (Y, σ).*

(2) *contra gΔ-continuous if f^{-1}(W) is a gΔ-closed set of (X, τ) for every Δ-open set W of (Y, σ).*

Proposition 5.4. *Every contra Δ-continuous is contra gΔ-continuous but not conversely.*

Proof. Let f : (X, τ) $\rightarrow$ (Y, σ) be a contra Δ-continuous map and let G be any Δ-open set in (Y, σ). Then, f^{-1}(G) is Δ-closed in X. Since every Δ-closed set is $g\Delta$-closed, f^{-1}(G) is $g\Delta$-closed in X. Therefore f is contra $g\Delta$-continuous. □

Example 5.1. *Let X and τ as in the Example 3.4. Let Y = $\{a, b, c\}$ and $\sigma = \{\phi, \{a\}, \{b\}, \{a, b\}, Y\}$. Then Δ-open sets are ϕ, $\{a\}$, $\{b\}$, $\{c\}$, $\{a, b\}$, $\{b, c\}$, Y. Define f : (X, τ) $\rightarrow$ (Y, σ) be the identity map. Then f is contra gΔ-continuous but not contra Δ-continuous, since $f^{-1}(\{a, b\}) = \{a, b\}$ is not Δ-closed in (X, τ).*

Remark 5.1. *The composition of two contra gΔ-continuous maps need not be contra gΔ-continuous.*

Example 5.2. *Let X and τ as in the Example 3.3. Let Y = $\{a, b, c\}$ and $\sigma = \{\phi, \{a\}, Y\}$. Then Δ-open sets are ϕ, $\{a\}$, $\{b, c\}$, Y ; gΔ-closed sets are power set of Y. Let Z = $\{a, b, c\}$ with $\gamma = \{\phi, \{a, c\}, Z\}$. Then Δ-open sets are ϕ, $\{b\}$, $\{a, c\}$, Z. Define f : (X, τ) $\rightarrow$ (Y, σ) and g : (Y, σ) $\rightarrow$ (Z, γ) be the identity maps. Clearly f and g are contra gΔ-continuous but their $g \circ f$: (X, τ) $\rightarrow$ (Z, γ) is not contra gΔ-continuous, because V = $\{b\}$ is Δ-open in (Z, γ) but ($g \circ f^{-1}(\{b\}) = f^{-1}(g^{-1}(\{b\})) = f^{-1}(\{b\}) = \{b\}$, which is not g$\Delta$-closed in (X, τ).*

Theorem 5.6. *Let f : (X, τ) $\rightarrow$ (Y, σ) be a map. Then the following conditions are equivalent*

(1) *f is contra g*Δ*-continuous.*

(2) *The inverse image of each* Δ*-open set in P is g*Δ*-closed in X.*

(3) *The inverse image of each* Δ*-closed set in P is g*Δ*-open in X.*

(4) *For each point x in X and each* Δ*-closed set G in P with f(x)* $\in$ *G, there is an g*Δ*-open set U in X containing x such that f(U)* $\subset$ *G.*

Proof. (1) $\Rightarrow$ (2). Let G be Δ-open in Y. Then Y $-$ G is Δ-closed in Y. By definition of contra $g\Delta$-continuous, f^{-1}(Y $-$ G) is $g\Delta$-open in X. But f^{-1}(Y $-$ G) = X $-$ f^{-1}(G). This implies f^{-1}(G) is $g\Delta$-closed in X.

(2) $\Rightarrow$ (3) Let G be any Δ-closed set in Y. Then Y $-$ G is Δ-open set in Y. By the assumption of (2), f^{-1}(Y $-$ G) is $g\Delta$-closed in X. But f^{-1}(Y $-$ G) = X $-$ f^{-1}(G). This implies f^{-1}(G) is $g\Delta$-open in X.

(3) $\Rightarrow$ (4). Let x $\in$ X and G be any Δ-closed set in Y with f(x) $\in$ G. By (3), f^{-1}(G) is $g\Delta$-open in X. Set U = f^{-1}(G). Then there is an $g\Delta$-open set U in X containing x such that f(U) $\subset$ G.

(4) $\Rightarrow$ (1). Let x $\in$ X and G be any Δ-closed set in Y with f(x) $\in$ G. Then Y $-$ G is Δ-open in Y with f(x) $\in$ G. By (4), there is an $g\Delta$-open set U in X containing x such that f(U) $\subset$ G. This implies U = f^{-1}(G). Therefore, X $-$ U = X $-$ f^{-1}(G) = f^{-1}(Y $-$ G) which is $g\Delta$-closed in X. □

Theorem 5.7. *Let f : (X,* τ*)* $\to$ *(Y,* σ*) and g : (Y,* σ*)* $\to$ *(Z,* γ*). Then the following properties hold:*

(1) *If f is contra g*Δ*-continuous and g is* Δ*-continuous then* $g \circ f$ *is contra g*Δ*-continuous.*

(2) *If f is contra g*Δ*-continuous and g is contra* Δ*-continuous then* $g \circ f$ *is g*Δ*-continuous.*

(3) *If f is g*Δ*-continuous and g is contra* Δ*-continuous then* $g \circ f$ *is contra g*Δ*-continuous.*

Proof. (1) Let G be Δ-closed set in Z. Since g is Δ-continuous, g^{-1}(G) is Δ-closed in Y. Since f is contra $g\Delta$-continuous, $(g \circ f)^{-1}$(G) = $f^{-1}(g^{-1}$(G)) is $g\Delta$-open in X. Therefore g $\circ$ f is contra $g\Delta$-continuous.

(2) Let G be any Δ-closed set in Z. Since g is contra Δ-continuous, g^{-1}(G) is Δ-open in Y. Since f is contra $g\Delta$-continuous, $(g \circ f)^{-1}$(G) = $f^{-1}(g^{-1}$(G)) is $g\Delta$-closed in X. Therefore g ∘ f is $g\Delta$-continuous.

(3) Let G be any Δ-closed set in Z. Since g is contra Δ-continuous, g^{-1}(G) is Δ-open in Y. Since f is $g\Delta$-continuous, $(g \circ f)^{-1}$(G) = $f^{-1}(g^{-1}$(G)) is $g\Delta$-open in X. Therefore g ∘ f is contra $g\Delta$-continuous. □

Theorem 5.8. *Let f : (X, τ) → (Y, σ) is gΔ-irresolute map and g : (Y, σ) → (Z, γ) is contra Δ-continuous map, then g ∘ f : (X, τ) → (Z, γ) is contra gΔ-continuous map.*

Proof. Since g is contra Δ-continuous from (Y, σ) → (Z, γ), for any Δ-open set in z as a subset of Z, we get, g^{-1}(z) = G is a Δ-closed set in (Y, σ). By Proposition 3.1, it implies that g^{-1}(z) = G is $g\Delta$-closed in (Y, σ). As f is $g\Delta$-irresolute map. We get $(g \circ f)^{-1}$(z) = $f^{-1}(g^{-1}$(z)) = f^{-1}(G) = S and S is a $g\Delta$-closed in (X, τ). Hence g ∘ f is a contra $g\Delta$-continuous map. □

Theorem 5.9. *Let f : (X, τ) → (Y, σ) is gΔ-irresolute map and g : (Y, σ) → (Z, γ) is contra gΔ-continuous map, then g ∘ f : (X, τ) → (Z, γ) is contra gΔ-continuous map.*

Proof. Since g is contra $g\Delta$-continuous from (Y, σ) → (Z, γ), for any Δ-open set in z as a subset of Z, we get, g^{-1}(z) = G is a $g\Delta$-closed set in (Y, σ). As f is $g\Delta$-irresolute map. We get $(g \circ f)^{-1}$(z)= $f^{-1}(g^{-1}$(z)) = f^{-1}(G) = S and S is a $g\Delta$-closed in (X, τ). Hence g ∘ f is a contra $g\Delta$-continuous map. □

Theorem 5.10. *Let f : (X, τ) → (Y, σ) be a map and g: (X, τ) → ((X, τ) × (Y, σ)) the graph map of f, defined by g(x) = (x, f(x)) for every x ∈ X. If g is contra gΔ-continuous, then f is contra gΔ-continuous.*

Proof. Let G be an Δ-open set in (Y, σ). Then ((X, τ) × G) is an Δ-open set in ((X, τ) × (Y, σ)). It follows from Theorem 5.6, that f^{-1}(G) = g^{-1}((X, τ) × G) is $g\Delta$-closed in (X, τ). Thus, f is contra $g\Delta$-continuous. □

6. **Generalized Δ-locally closed sets**

We introduce the following definitions.

Definition 6.1. *A subset A of an topological space (X, τ) is called an Δ-locally closed (briefly, Δ-lc) sets if $A = S \cap G$ where S is Δ-open and G is Δ-closed.*
The class of all Δ-locally closed sets in a topological space (X, τ) is denoted by $\Delta LC(X)$.

Definition 6.2. *A subset A of an topological space (X, τ) is called an generalized Δ-locally closed (briefly, $g\Delta$-lc) sets if $A = E \cap F$ where E is $g\Delta$-open and F is $g\Delta$-closed.*
The class of all $g\Delta$-locally closed sets in topological spaces (X, τ) is denoted by $G\Delta LC(X)$.

Proposition 6.1. *Every Δ-closed (resp. Δ-open) set is Δ-lc-set but not conversely.*

Proof. It follows from Definition 6.1. □

Example 6.1. *Let X and τ as in the Example 3.1. Then Δ-open sets are ϕ, $\{c\}$, $\{a, b\}$, X ; Δlc-sets are ϕ, $\{c\}$, $\{a, b\}$, $\{c, d\}$, $\{a, b, d\}$, X. Here, the set $\{a, b\}$ is Δ-lc set but it is not Δ-closed and the set $\{a, b, d\}$ is Δ-lc set but it is not Δ-open in (X, τ).*

Proposition 6.2. *Every $g\Delta$-closed (resp. $g\Delta$-open) set is $g\Delta$-lc-set but not conversely.*

Proof. It follows from Definition 6.2. □

Example 6.2. *Let X and τ as int the Example 3.3. Then $g\Delta$-open sets are ϕ, $\{a\}$, $\{b\}$, $\{c\}$, $\{a, b\}$, $\{b, c\}$, X; $g\Delta$-lc-sets are power set of X. Here, the set $\{b\}$ is $g\Delta$-lc set but it is not $g\Delta$-closed and the set $\{a, c\}$ is $g\Delta$-lc set but it is not $g\Delta$-open in (X, τ).*

Proposition 6.3. *Every Δ-lc-set is $g\Delta$-lc-set but not conversely.*

Proof. It follows from Proposition 3.1 and 3.5. □

Example 6.3. *Let X, τ as in the Example 3.4. Then Δ-lc-sets are ϕ, $\{a\}$, $\{b, c\}$, X ; $g\Delta$-lc-sets are power set of X. Here, the set $\{a, b\}$ is $g\Delta$-lc set but it is not Δ-lc set in (X, τ).*

Theorem 6.1. *Let (X, τ) be a $T\Delta 1/2$-space. Then $G\Delta LC(X) = \Delta LC(X)$.*

Proof. Since every $g\Delta$-open set is Δ-open and every $g\Delta$-closed set is Δ-closed in (X, τ), GΔLC(X) $\subseteq$ ΔLC(X) and hence GΔLC(X) = ΔLC(X). □

Definition 6.3. *A subset A of a space (X, τ) is called*

(1) *$g\Delta$-lc*-set if $A = O \cap P$, where O is $g\Delta$-open in (X, τ) and P is Δ-closed in (X, τ).*
(2) *$g\Delta$-lc**-set if $A = R \cap S$, where R is Δ-open in (X, τ) and S is $g\Delta$-closed in (X, τ).*

The class of all $g\Delta$-lc* (resp. $g\Delta$-lc**) sets in a topological space (X, τ) is denoted by $G\Delta LC^*(X)$ (resp. $G\Delta LC^{**}(X)$).

Proposition 6.4. *Every Δ-lc-set is $g\Delta$-lc*-set but not conversely.*

Proof. It follows from Definitions 6.1 and 6.3 (1). □

Example 6.4. *Let X and τ as in the Example 6.1. Then $g\Delta$-lc*-sets are ϕ, $\{a\}$, $\{b\}$, $\{c\}$, $\{d\}$, $\{a, b\}$, $\{a, c\}$, $\{a, d\}$, $\{b, c\}$, $\{b, d\}$, $\{c, d\}$, $\{a, b, c\}$, $\{a, b, d\}$, X. Here, the set $\{a, b, c\}$ is $g\Delta$-lc*-set but it is not Δ-lc-set in (X, τ).*

Proposition 6.5. *Every Δ-lc-set is $g\Delta$-lc**-set but not conversely.*

Proof. It follows from Definitions 6.1 and 6.3(2). □

Example 6.5. *Let X and τ as in the Example 6.1. Then $g\Delta$-lc**-sets are power set of x. Here, the set $\{b, c, d\}$ is $g\Delta$-lc**-set but it is notΔ-lc-set in (X, τ).*

Proposition 6.6. *Every $g\Delta$-lc*-set is $g\Delta$-lc-set but not conversely.*

Proof. It follows from Definitions 6.2 and 6.3(1). □

Example 6.6. *Let X and τ as in the Example 6.4. Then $g\Delta$-lc-sets are power sets of X. Here, the set $\{a, c, d\}$ is $g\Delta$-lc-set but it is not $g\Delta$-lc*-set in (X, τ).*

Proposition 6.7. *Every $g\Delta$-lc**-set is $g\Delta$-lc-set but not conversely.*

Proof. It follows from Definitions 6.2 and 6.3 (2). □

Question 1. Give an example for a set which is $g\Delta$-lc-set but not $g\Delta$-lc**-set.

The following results are characterizations of $g\Delta$-lc-sets, $g\Delta$-lc*-sets and $g\Delta$-lc**-sets.

Theorem 6.2. *For a subset A of (X, τ) the following statements are equivalent:*

(1) $A \in G\Delta LC(X)$,

(2) $A = S \cap g\Delta\text{-}cl(A)$ *for some* $g\Delta$*-open set S,*

(3) $g\Delta\text{-}cl(A) - A$ *is* $g\Delta$*-closed,*

(4) $A \cup (g\Delta\text{-}cl(A))^c$ *is* $g\Delta$*-open,*

(5) $A \subseteq g\Delta\text{-}int(A \cup (g\Delta\text{-}cl(A))^c)$.

Proof. (1) $\Rightarrow$ (2). Let A $\in$ GΔLC(X). Then A = S $\cap$ G where S is $g\Delta$-open and G is $g\Delta$-closed. Since A $\subseteq$ G, $g\Delta$-cl(A) $\subseteq$ G and so S $\cap$ $g\Delta$-cl(A) $\subseteq$ A. Also A $\subseteq$ S and A $\subseteq$ $g\Delta$-cl(A) implies A $\subseteq$ S $\cap$ $g\Delta$-cl(A) and therefore A = S $\cap$ $g\Delta$-cl(A).

(2) $\Rightarrow$ (3). A = S $\cap$ $g\Delta$-cl(A) implies $g\Delta$-cl(A) $-$ A = $g\Delta$-cl(A) $\cap$ S^c which is $g\Delta$-closed since S^c is $g\Delta$-closed and $g\Delta$-cl(A) is $g\Delta$-closed.

(3) $\Rightarrow$ (4). A $\cup$ ($g\Delta$-cl(A))^c = ($g\Delta$-cl(A) $-$ A)^c and by assumption, ($g\Delta$-cl(A) $-$ A)^c is $g\Delta$-open and so is A $\cup$ ($g\Delta$-cl(A))^c.

(4) $\Rightarrow$ (5). By assumption, A $\cup$ ($g\Delta$-cl(A))^c = $g\Delta$-int(A $\cup$ ($g\Delta$-cl(A))^c) and hence A $\subseteq$ $g\Delta$-int(A $\cup$ ($g\Delta$-cl(A))^c).

(5) $\Rightarrow$ (1). By assumption and since A $\subseteq$ $g\Delta$-cl(A), A = $g\Delta$-int(A $\cup$ ($g\Delta$-cl(A))^c) $\cap$ $g\Delta$-cl(A). Therefore, A $\in$ $G\Delta$LC(X). □

Theorem 6.3. *For a subset A of (X, τ), the following statements are equivalent:*

(1) $A \in G\Delta LC^*(U)$,

(2) $A = S \cap \Delta cl(A)$ *for some* $g\Delta$*-open set S,*

(3) $\Delta cl(A) - A$ *is* $g\Delta$*-closed,*

(4) $A \cup (\Delta cl(A))^c$ *is* $g\Delta$*-open.*

Proof. (1) ⇒ (2). Let A $\in G\Delta$LC *(U). There exist an $g\Delta$-open set S and a Δ-closed set G such that A = S $\cap$ G. Since A $\subseteq$ S and A $\subseteq$ Δcl(A), A $\subseteq$ S $\cap$ Δcl(A). Also since Δcl(A) $\subseteq$ G, S $\cap$ Δcl(A) $\subseteq$ S $\cap$ G = A. Therefore A = S $\cap$ Δcl(A).

(2) ⇒ (1). Since S is $g\Delta$-open and Δcl(A) is a Δ-closed set, A = S $\cap$ Δcl(A) $\in G\Delta$LC *(U).

(2) ⇒ (3). Since Δcl(A) − A = Δcl(A) $\cap$ S^c, Δcl(A) − A is $g\Delta$-closed since S^c is $g\Delta$-closed.

(3) ⇒ (2). Let S = (Δcl(A) − A)^c. Then by assumption S is $g\Delta$-open in (X, τ) and A = S $\cap$ Δcl(A).

(3) ⇒ (4). Let G = Δcl(A) − A. Then G^c = A $\cup$ (Δcl(A))^c and A $\cup$ (Δcl(A))^c is $g\Delta$-open.

(4) ⇒ (3). Let S = A $\cup$ (Δcl(A))^c. Then S^c is $g\Delta$-closed and S^c = Δcl(A) − A and so Δcl(A) − A is $g\Delta$-closed. □

Theorem 6.4. *Let A be a subset of (X, τ). Then A $\in G\Delta LC^{**}$(X) if and only if A = S $\cap$ $g\Delta$-cl(A) for some Δ-open set S.*

Proof. (1) ⇒ (2). Let A $\in G\Delta$LC**(X). Then A = S $\cap$ G where S is Δ-open and G is $g\Delta$-closed. Since A $\subseteq$ G, $g\Delta$-cl(A) $\subseteq$ G. We obtain A = A $\cap$ $g\Delta$-cl(A) = S $\cap$ G $\cap$ $g\Delta$-cl(A) =S $\cap$ $g\Delta$-cl(A).

(2) ⇒ (1). Since S is Δ-open and $g\Delta$-cl(A) is a $g\Delta$-closed set, A = S $\cap$ $g\Delta$-cl(A) $\in$ $G\Delta$LC**(X). □

Corollary 6.1. *Let A be a subset of (X, τ). If A $\in G\Delta LC^{**}$(X), then $g\Delta cl$(A) − A is $g\Delta$-closed and A $\cup$ ($g\Delta$-cl(A))^c is $g\Delta$-open.*

Proof. Let A $\in G\Delta$LC**(X). Then by Theorem 6.4, A = S $\cap$ $g\Delta$-cl(A) for some Δ-open set S and $g\Delta$-cl(A) − A = $g\Delta$-cl(A) $\cap$ S^c is $g\Delta$-closed in (X, τ). If G = $g\Delta$-cl(A) − A, then G^c = A $\cup$ ($g\Delta$-cl(A))^c and G^c is $g\Delta$-open and so is A $\cup$ ($g\Delta$-cl(A))^c. □

7. $G\Delta$LC -Continuous and $G\Delta$LC -Irresolute maps

We introduce the following definitions.

Definition 7.1. *A map $f : (X, \tau) \to (Y, \sigma)$ is said to be Δ locally closed-continuous (briefly, ΔLC-continuous) if $f^{-1}(V)$ is ΔLC-set in (X, τ) for every Δ-open set V of (Y, σ).*

Definition 7.2. *A map $f : (X, \tau) \to (Y, \sigma)$ is said to be $G\Delta LC$-continuous (resp. $G\Delta LC^*$-continuous, $G\Delta LC^{**}$-continuous) if $f^{-1}(V)$ is $G\Delta LC$-set (resp. $G\Delta LC^*$-set, $G\Delta LC^{**}$-set) in (X, τ) for every Δ-open set V of (Y, σ).*

Theorem 7.1. *Let $f : (X, \tau) \to (Y, \sigma)$ be a map. Then*

(1) *If f is Δ-continuous, then it is ΔLC-continuous.*
(2) *If f is Δ-continuous, then it is $G\Delta LC$-continuous.*
(3) *If f is $g\Delta$-continuous, then it is $G\Delta LC$-continuous.*

Proof. (1) It is an immediate consequence of Proposition 6.1.

(2) It is an immediate consequence of Proposition 6.1 and 6.3.

(3) It is an immediate consequence of Propositions 6.2. □

Theorem 7.2. *Let $f : (X, \tau) \to (Y, \sigma)$ be a map. Then*

(1) *If f is ΔLC-continuous, then it is $G\Delta LC$-continuous.*
(2) *If f is ΔLC-continuous, then it is $G\Delta LC^*$-continuous.*
(3) *If f is ΔLC-continuous, then it is $G\Delta LC^{**}$-continuous.*
(4) *If f is $G\Delta LC^*$-continuous, then it is $G\Delta LC$-continuous.*
(5) *If f is $G\Delta LC^{**}$-continuous, then it is $G\Delta LC$-continuous.*

Proof. (1) It is an immediate consequence of Proposition 6.3.

(2) It is an immediate consequence of Proposition 6.4.

(3) It is an immediate consequence of Propositions 6.5.

(4) It is an immediate consequence of Propositions 6.6.

(5) It is an immediate consequence of Propositions 6.7. □

Definition 7.3. *A map f : (X, τ) $\rightarrow$ (Y, σ) is said to be $G\Delta LC$ -irresolute (resp. $G\Delta LC$ *-irresolute, $G\Delta LC^{**}$-irresolute) if $f^{-1}(V)$ is $G\Delta LC$ -set (resp. $G\Delta LC^*$-set, $G\Delta LC^{**}$-set) in (X, τ) for every $G\Delta LC$ -set (resp. $G\Delta LC^*$-set, $G\Delta LC^{**}$-set) V of (Y, σ).*

Theorem 7.3. *Let f : (X, τ) $\rightarrow$ (Y, σ) be a map. Then*

(1) *If f is $G\Delta LC$ -irresolute then it is $G\Delta LC$ -continuous.*

(2) *If f is $G\Delta LC^*$-irresolute then it is $G\Delta LC^*$-continuous.*

(3) *If f is $G\Delta LC^{**}$-irresolute then it is $G\Delta LC^{**}$-continuous.*

Proof. (1) Let f : (X, τ) $\rightarrow$ (Y, σ) be a $G\Delta$LC -irresolute map. Let V be a Δ-open set of (Y, σ). Since every Δ-open set is $g\Delta$-open and $g\Delta$-open set is $g\Delta$-lc-set [by the Proposition 3.5 and Theorem 6.2], V is $G\Delta$LC -set of (Y, σ). Since f is $G\Delta$LC -irresolute, then $f^{-1}(V)$ is a $G\Delta$LC -set of (X, τ). Therefore f is $G\Delta$LC -continuous.

(2) Let f : (X, τ) $\rightarrow$ (Y, σ) be a $G\Delta$LC *-irresolute map. Let V be a Δ-open set of (Y, σ). Since every Δ-open set is Δ-lc set and Δ-lc-set is $g\Delta$-lc*-set [by Proposition 6.1 and Proposition 6.4], V is $G\Delta$LC *-set of (Y, σ). Since f is $G\Delta$LC *-irresolute, then $f^{-1}(V)$ is a $G\Delta$LC *-set of (X, τ). Therefore f is $G\Delta$LC *-continuous.

(3) Let f : (X, τ) $\rightarrow$ (Y, σ) be a $G\Delta$LC **-irresolute map. Let V be a Δ-open set of (Y, σ). Since every Δ-open set is Δ-lc-set and Δ-lc-set is $g\Delta$-lc**-set [by Proposition 6.1 and Proposition 6.5], A is $G\Delta$LC **-set of (Y, σ). Since f is $G\Delta$LC **-irresolute, then $f^{-1}(V)$ is a $G\Delta$LC **-set of (X, τ). Therefore f is $G\Delta$LC **-continuous. □

Theorem 7.4. *Let f : (X, τ) $\rightarrow$ (Y, σ) and g : (Y, σ) $\rightarrow$ (Z, γ) be any two maps. Then*

(1) *$g \circ f$ is $G\Delta LC$-continuous if g is Δ-continuous and f is $G\Delta LC$-continuous.*

(2) *$g \circ f$ is $G\Delta LC$-irresolute if both f and g are $G\Delta LC$-irresolute.*

(3) *$g \circ f$ is $G\Delta LC$-continuous if g is $G\Delta LC$-continuous and f is $G\Delta LC$-irresolute.*

Proof. (1) Since g is a Δ-continuous from (Y, σ) $\rightarrow$ (Z, γ), for any Δ-open set z as a subset of Z, we get $g^{-1}(z)$ = G is a Δ-open set in (Y, σ) . As f is a $G\Delta$LC-continuous map. We get $(g \circ f)^{-1}(z) = f^{-1}(g^{-1}(z)) = f^{-1}(G)$ = S and S is a $G\Delta$LC-set in (X, τ), since every Δ-open set is $g\Delta$-open set and $g\Delta$-open set is $g\Delta$-lc-set [by the Proposition 3.5 and Proposition 6.2]. Hence (g ∘ f) is a $G\Delta$LC-continuous map.

(2) Consider two $G\Delta$LC-irresolute maps, f : (X, τ) → (Y, σ) and g : (Y, σ) → (Z, γ) is a $G\Delta$LC-irresolute maps. As g is consider to be a $G\Delta$LC-irresolute map, by Definition 7.3, for every $g\Delta$-lc-set z ⊆ (Z, γ), $g^{-1}(z)$ = G is a $g\Delta$-lc-set in (Y, σ). Again since f is $G\Delta$LC-irresolute, $(g \circ f)^{-1}(z) = f^{-1}(g^{-1}(z)) = f^{-1}(G)$ = S and S is a $g\Delta$-lc-set in (X, τ). Hence (g ∘ f) is a $G\Delta$LC-irresolute map.

(3) Let g be a $G\Delta$LC-continuous map from (Y, σ) → (Z, γ) and z subset of Z be a Δ-open set. Therefore $g^{-1}(z)$ = G is a $g\Delta$-lc-set in (Y, σ), since every Δ-open set is $g\Delta$-open set and $g\Delta$-open set is $g\Delta$-lc-set [by the Proposition 3.5 and Proposition 6.2]. Also since f is $G\Delta$LC-irresolute, we get $(g \circ f)^{-1}(z) = f^{-1}(g^{-1}(z)) = f^{-1}(G)$ = S and S is a $g\Delta$-lc-set in (X, τ). Hence (g ∘ f) is a $G\Delta$LC-continuous map. □

Conclusion

Every year different type of topological spaces are introduced by many topologist. Now a days available topologies are ideal topology, nano topology, nano ideal topology, micro topology, micro ideal topology, micro grill topology and so on. In this paper, we introduced and studied α-Δ-open sets in topological spaces. We offer a new class of sets called $g\Delta$-closed sets in topological spaces and we study some of its basic properties. we introduce $g\Delta$-interior and $g\Delta$-closure and study some of its basic properties. We introduce $g\Delta$-continuous maps and $g\Delta$-irresolute maps. we introduce the classes of Δ-lc-set, $g\Delta$-lc-set, $g\Delta$-lc*-sets, $g\Delta$-lc**-sets and study some of its basic properties. Finally we introduced and studied ΔLC-continuous, $G\Delta$LC-continuous map and $G\Delta$LC-irresolute map. In future, we have extended this work in various topological fields with some applications.

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