SUM CORDIAL LABELING FOR \( n \)-STAR GRAPH

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**Abstract.** In this paper, we proved that \( n \)-star graph with or without wedge is sum cordial graph.

**Keyword:** Star graph, Sum Cordial graph and Wedge.

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1. **Introduction**

In [1, 2], we considered undirected, finite and simple graph \( G = (V(G), E(G)) \). In [3], they proved some graphs like path \( P_n \), cycle \( C_n \), star \( K_{1,n} \), bistar \( B_{n,n} \) are Sum cordial graphs. We provided some definitions which are used for our present study.

**Definition 1.1.** Wedge

When a disconnected graph’s are connected by an edge in order to form a single connected graph is known as wedge. It is denoted by the symbol \( \wedge \), \( \omega(G\wedge) < \omega(G) \).

**Definition 1.2.** Sum cordial graph

A binary vertex labeling of a graph \( G \) with induced edge labeling \( z^* : E(G) \rightarrow \{0, 1\} \) defined by \( z^*(uv) = (f(u) + f(v))(mod\ 2) \) is called sum cordial labeling if \( |v_f(0) - v_f(1)| \leq 1 \) and \( |e_f(0) - e_f(1)| \leq 1 \). A graph \( G \) is sum cordial if it admits sum cordial labeling.

2. **Results**

**Theorem 2.1.** Any two star graph with wedge is sum cordial graph.
Proof. Let the graph $G = K_{1,v} \wedge K_{1,y}$. Let $V(G)$ be node and $E(G)$ be edge set of graph.

Then,

$V(G) = \{m, c\} \cup \{m_\gamma; 1 \leq \gamma \leq v\} \cup \{c_\gamma; 1 \leq \gamma \leq y\}.

E(G) = \{mm_\gamma; 1 \leq \gamma \leq v\} \cup \{cc_\gamma; 1 \leq \gamma \leq y\} \cup \{m_\gamma c_\gamma\}.

Then $G$ has $v + y + 2$ nodes and $v + y + 1$ edges. Now to prove that $G$ is sum cordial graph for all $v > 1$ and $y > 1$.

Case (i) : When $v < y$.

Subcase (a) : Suppose $v$ is odd and $y$ is odd.

The node and link labeling of $G$ is,

$z : V(G) \to \{0, 1\}$ and $z* : E(G) \to \{0, 1\}$.

Defining the node labeling of $G$ as,

$z(m) = 0 ; z(c) = 1.$

$z(m_{2\gamma - 1}) = 1$ for $1 \leq \gamma \leq \left\lfloor \frac{v}{2} \right\rfloor$.

$z(m_\gamma) = 0$ for $1 \leq \gamma \leq \left\lfloor \frac{v}{2} \right\rfloor$.

$z(c_{2\gamma - 1}) = 0$ for $1 \leq \gamma \leq \left\lfloor \frac{y}{2} \right\rfloor$.

$z(c_\gamma) = 1$ for $1 \leq \gamma \leq \left\lfloor \frac{y}{2} \right\rfloor$.

Thus the edge labelings are given by,

$mm_{2\gamma - 1} = 1$ for $1 \leq \gamma \leq \left\lfloor \frac{v}{2} \right\rfloor$.

$mm_\gamma = 0$ for $1 \leq \gamma \leq \left\lfloor \frac{v}{2} \right\rfloor$.

$cc_{2\gamma - 1} = 1$ for $1 \leq \gamma \leq \left\lfloor \frac{y}{2} \right\rfloor$.

$cc_\gamma = 0$ for $1 \leq \gamma \leq \left\lfloor \frac{y}{2} \right\rfloor$.

Then, wedge is labelled by $m_{2\gamma - 1}c_\gamma = 0$.

Hence, $|v_z(0) - v_z(1)| \leq 1$.

$|e_z(0) - e_z(1)| \leq 1$.

Subcase (b) : Suppose $v$ is even and $y$ is even.

Defining the node labeling of $G$ as,

$z(m) = 0 ; z(c) = 1.$
\[ z(m_{2\gamma-1}) = 1 \quad \text{for} \quad 1 \leq \gamma \leq \frac{v}{2}. \]
\[ z(m_{2\gamma}) = 0 \quad \text{for} \quad 1 \leq \gamma \leq \frac{v}{2}. \]
\[ z(c_{2\gamma-1}) = 0 \quad \text{for} \quad 1 \leq \gamma \leq \frac{y}{2}. \]
\[ z(c_{2\gamma}) = 1 \quad \text{for} \quad 1 \leq \gamma \leq \frac{y}{2}. \]

Thus the edge labelings are given by,
\[ mm_{2\gamma-1} = 1 \quad \text{for} \quad 1 \leq \gamma \leq \frac{v}{2}. \]
\[ mm_{2\gamma} = 0 \quad \text{for} \quad 1 \leq \gamma \leq \frac{v}{2}. \]
\[ cc_{2\gamma-1} = 1 \quad \text{for} \quad 1 \leq \gamma \leq \frac{y}{2}. \]
\[ cc_{2\gamma} = 0 \quad \text{for} \quad 1 \leq \gamma \leq \frac{y}{2}. \]

Then, wedge is labelled by \( m_{2\gamma-1}c_{2\gamma-1} = 1 \).

Hence, \(|v_z(0) - v_z(1)| \leq 1.\]
\(|e_z(0) - e_z(1)| \leq 1.\]

**Case (ii) :** When \( v = y.\)

**Subcase (a) :** Suppose \( v \) is odd and \( y \) is odd.

The node and link labeling of \( G \) is,
\[ z : V(G) \to \{0, 1\} \quad \text{and} \quad z* : E(G) \to \{0, 1\}. \]

Defining the node labeling of \( G \) as,
\[ z(m) = 0 ; z(c) = 0. \]
\[ z(m_{2\gamma-1}) = 1 \quad \text{for} \quad 1 \leq \gamma \leq \left\lfloor \frac{v}{2} \right\rfloor. \]
\[ z(m_{2\gamma}) = 0 \quad \text{for} \quad 1 \leq \gamma \leq \left\lfloor \frac{v}{2} \right\rfloor. \]
\[ z(c_{2\gamma-1}) = 1 \quad \text{for} \quad 1 \leq \gamma \leq \left\lfloor \frac{y}{2} \right\rfloor. \]
\[ z(c_{2\gamma}) = 0 \quad \text{for} \quad 1 \leq \gamma \leq \left\lfloor \frac{y}{2} \right\rfloor. \]

Thus the edge labelings are given by,
\[ mm_{2\gamma-1} = 1 \quad \text{for} \quad 1 \leq \gamma \leq \left\lfloor \frac{v}{2} \right\rfloor. \]
\[ mm_{2\gamma} = 0 \quad \text{for} \quad 1 \leq \gamma \leq \left\lfloor \frac{v}{2} \right\rfloor. \]
\[ cc_{2\gamma-1} = 1 \quad \text{for} \quad 1 \leq \gamma \leq \left\lfloor \frac{y}{2} \right\rfloor. \]
\[ cc_{2\gamma} = 0 \quad \text{for} \quad 1 \leq \gamma \leq \left\lfloor \frac{y}{2} \right\rfloor. \]
Then, wedge is labelled by $m_{2\gamma-1}c_{2\gamma-1} = 0$.

Hence, $|v_z(0) - v_z(1)| \leq 1$.

$|e_z(0) - e_z(1)| \leq 1$.

**Subcase (b)**: Suppose $v$ is even and $y$ is even.

Defining the node labeling of $G$ as,

- $z(m) = 0$ ; $z(c) = 1$.
- $z(m_{2\gamma-1}) = 1$ for $1 \leq \gamma \leq \frac{v}{2}$.
- $z(m_{2\gamma}) = 0$ for $1 \leq \gamma \leq \frac{v}{2}$.
- $z(c_{2\gamma-1}) = 0$ for $1 \leq \gamma \leq \frac{y}{2}$.
- $z(c_{2\gamma}) = 1$ for $1 \leq \gamma \leq \frac{y}{2}$.

Thus the edge labelings are given by,

- $mm_{2\gamma-1} = 1$ for $1 \leq \gamma \leq \frac{v}{2}$.
- $mm_{2\gamma} = 0$ for $1 \leq \gamma \leq \frac{v}{2}$.
- $cc_{2\gamma-1} = 1$ for $1 \leq \gamma \leq \frac{y}{2}$.
- $cc_{2\gamma} = 0$ for $1 \leq \gamma \leq \frac{y}{2}$.

Then, wedge is labelled by $m_{2\gamma-1}c_{2\gamma-1} = 1$.

Hence, $|v_z(0) - v_z(1)| \leq 1$.

$|e_z(0) - e_z(1)| \leq 1$.

Hence, the graph $G$ with wedge is sum cordial graph. \(\square\)

**Corollary 2.2.** Any two star graph $K_{1,v} \cup K_{1,t}$ is sum cordial graph.

**Proof.** This proof is similar to that of theorem 2.1. \(\square\)

**Theorem 2.3.** Any three star graph with wedge is sum cordial graph.

**Proof.** Let the graph $G = K_{1,v} \wedge K_{1,y} \wedge K_{1,t}$. Let $V(G)$ be node and $E(G)$ be edge set of graph.

Then,

- $V(G) = \{m, c, u\} \cup \{m_\gamma; 1 \leq \gamma \leq v\} \cup \{c_\gamma; 1 \leq \gamma \leq y\} \cup \{u_\gamma; 1 \leq \gamma \leq t\}$.
- $E(G) = \{mm_\gamma; 1 \leq \gamma \leq v\} \cup \{cc_\gamma; 1 \leq \gamma \leq y\} \cup \{uu_\gamma; 1 \leq \gamma \leq t\} \cup \{m_\gamma c_\gamma\}$
\[ \cup \{ c \gamma u_2 \gamma \} \]
Then G has \( v + y + t + 3 \) nodes and \( v + y + t + 2 \) edges. Now to prove that G is sum cordial graph for all \( v > 1, y > 1 \) and \( t > 1 \).

**Case (i)**: When \( v < y < t \).

**Subcase (a)**: Suppose \( v, y \) and \( t \) is odd.

The node and link labeling of G is ,
\[ z : V(G) \to \{0, 1\} \] and \[ z* : E(G) \to \{0, 1\} \].

Defining the node labeling of G as,
\[ z(m) = 0 ; z(c) = 1 \text{ and } z(u) = 0. \]
\[ z(m_{2\gamma - 1}) = 1 \text{ for } 1 \leq \gamma \leq \left\lceil \frac{v}{2} \right\rceil. \]
\[ z(m_{2\gamma}) = 0 \text{ for } 1 \leq \gamma \leq \left\lfloor \frac{v}{2} \right\rfloor. \]
\[ z(c_{2\gamma - 1}) = 0 \text{ for } 1 \leq \gamma \leq \left\lfloor \frac{y}{2} \right\rfloor. \]
\[ z(c_{2\gamma}) = 1 \text{ for } 1 \leq \gamma \leq \left\lfloor \frac{y}{2} \right\rfloor. \]
\[ z(u_{2\gamma - 1}) = 1 \text{ for } 1 \leq \gamma \leq \left\lfloor \frac{t}{2} \right\rfloor. \]
\[ z(u_{2\gamma}) = 0 \text{ for } 1 \leq \gamma \leq \left\lfloor \frac{t}{2} \right\rfloor. \]

Thus the edge labelings are given by,
\[ mm_{2\gamma - 1} = 1 \text{ for } 1 \leq \gamma \leq \left\lceil \frac{v}{2} \right\rceil. \]
\[ mm_{2\gamma} = 0 \text{ for } 1 \leq \gamma \leq \left\lfloor \frac{v}{2} \right\rfloor. \]
\[ cc_{2\gamma - 1} = 1 \text{ for } 1 \leq \gamma \leq \left\lceil \frac{y}{2} \right\rceil. \]
\[ cc_{2\gamma} = 0 \text{ for } 1 \leq \gamma \leq \left\lfloor \frac{y}{2} \right\rfloor. \]
\[ uu_{2\gamma - 1} = 1 \text{ for } 1 \leq \gamma \leq \left\lfloor \frac{t}{2} \right\rfloor. \]
\[ uu_{2\gamma} = 0 \text{ for } 1 \leq \gamma \leq \left\lfloor \frac{t}{2} \right\rfloor. \]

Then, wedge is labelled by \( m_{2\gamma}c_{2\gamma - 1} = 0. \)
And, wedge is labelled by \( c_{2\gamma - 1}u_{2\gamma} = 0. \)

Hence, \(|v_2(0) - v_2(1)| \leq 1.\)
\(|e_2(0) - e_2(1)| \leq 1.\)

Similarly, the following subcases also stands true for this theorem.
Subcase (b) : Suppose v,y and t is even then it is sum cordial graph.
Subcase (c) : Suppose v and y are odd, t is even then it is sum cordial graph.
Subcase (d) : Suppose v and y are even, t is odd then it is sum cordial graph.
Subcase (e) : Suppose v and t are odd, y is even then it is sum cordial graph.
Subcase (f) : Suppose v and t are even, y is odd then it is sum cordial graph.

Therefore, the following cases are also sum cordial labeling.

Case (ii) : When \( v = y < t \)
Case (iii) : When \( v > y < t \)

Hence, the graph G with wedge is sum cordial graph.

Corollary 2.4. Any three star graph \( K_{1,v} \cup K_{1,y} \cup K_{1,t} \) is sum cordial graph.

Proof. This proof is similar to that of theorem 2.3.

Corollary 2.5. Any four and more star graph with or without wedge is sum cordial graph.

Proof. This proof is also similar to that of theorem 2.3.

Corollary 2.6. Any n-star graph with or without wedge is sum cordial graph.

Proof. This proof is obvious. If n copies of star graph is connected by an wedge then \( n(K_{1,n}) \) is sum cordial graph. Therefore, we conclude that n - star graph with wedge is sum cordial graph. Similarly, n copies of star graph without wedge is also sum cordial graph i.e., \( n(\cup K_{1,n}) \). Hence, we conclude that n - star graph without wedge is sum cordial graph.

Applications of graph labeling

A static network are often described as a particular graph by connecting nodes in some topology and labeling will be applied for automatic routing of information in an exceedingly network. The graph are often cycle, path, circuit, star, walk, connected that represent a set network. For every network
labeling is completed with a relentless that helps routing to mechanically find next node within the network.

CONCLUSION

In this paper, we have proved that every two star and three graph is sum cordial labeling and hence we concluded n-star graph is also sum cordial labeling. Future work will be carried out on finding the bounds for different types of labeling techniques and to generalize for n star graph.

REFERENCES