

ON $n\mathcal{I}_{\tilde{g}}$ -HOMEOMORPHISM IN NANO IDEAL TOPOLOGICAL SPACES

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ABSTRACT. In this paper, we introduce and study two new homeomorphism namely $n\mathcal{I}_{\tilde{g}}$ -homeomorphism and $*n\mathcal{I}_{\tilde{g}}$ -Homeomorphism in nano ideal topological spaces.

1. Introduction

In 1990, Jankovic and Hamlett [[7], [8]] have considered the local function in ideal topological space any they have obtained a new topology. In 2016, Paimala et al [10] introduce a similar type with the local function in nano topological spaces. Before starting the discussion we shall consider the following concepts. Let $(O, \mathcal{N}, \mathcal{I})$ be an nano ideal

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topological space with an ideal \mathcal{I} on O , where $\mathcal{N} = \tau_R(X)$ and $(\cdot)_n^* : \wp(O) \rightarrow \wp(O)$ ($\wp(O)$ is the set of all subsets of O) [10, 11]. For a subset $A \subseteq O$, $A_n^*(\mathcal{I}, \mathcal{N}) = \{o \in O : G_n \cap A \notin \mathcal{I}, \text{ for every } G_n \in G_n(o)\}$, where $G_n(o) = \{G_n \mid o \in G_n, G_n \in \mathcal{N}\}$ is called the nano local function (briefly n-local function) of A with respect to \mathcal{I} and \mathcal{N} . We will simply write A_n^* for $A_n^*(\mathcal{I}, \mathcal{N})$. Parimala et al [11] introduced the concept of nano ideal generalized closed sets in nano ideal topological spaces and investigated some of its basic properties. Recently, Ganesan [5] introduced and studied $n\mathcal{I}_g$ -continuous map and $n\mathcal{I}_g$ -irresolute map in nano ideal topological spaces. In this paper, we introduced the concept of $n\mathcal{I}_g$ -closed maps, $n\mathcal{I}_g$ -open maps, $n\mathcal{I}_g$ -Homeomorphism in nano ideal topological spaces and study its relationship with existing homeomorphisms. A new class of maps $^*n\mathcal{I}_g$ -Homeomorphism is introduced which from a subclass of $n\mathcal{I}_g$ -Homeomorphisms. we establish that the set of all $^*n\mathcal{I}_g$ -Homeomorphism $(U, \mathcal{N}, \mathcal{I})$ onto itself is a group under the composition of maps.

2. Preliminaries

Definition 2.1. [10, 11] A subset A of a nano ideal topological space $(U, \mathcal{N}, \mathcal{I})$ is said to be $n\star$ -closed if $A_n^* \subseteq A$.

Lemma 2.1. [10, 11] Let $(U, \mathcal{N}, \mathcal{I})$ be an nano topological space with an ideal \mathcal{I} and $A \subseteq A_n^*$, then $A_n^* = n-cl(A_n^*) = n-cl(A)$

Definition 2.2. A subset A of an nano ideal topological space $(U, \mathcal{N}, \mathcal{I})$ is said to be

- (1) nano- \mathcal{I} -generalized closed (briefly, $n\mathcal{I}_g$ -closed) [11] if $A_n^* \subseteq V$ whenever $A \subseteq V$ and V is n -open.
- (2) $n\mathcal{I}_g$ -open if its complement is $n\mathcal{I}_g$ -closed.
- (3) $n\mathcal{I}_g$ -closed [3] if $A_n^* \subseteq U$ whenever $A \subseteq U$ and U is nsg -open,
- (4) $n\mathcal{I}_g$ -open if its complement is $n\mathcal{I}_g$ -closed.

Definition 2.3. [9] A map $f : (K, \mathcal{N}, \mathcal{I}) \rightarrow (L, \mathcal{N}', \mathcal{J})$ is said to be $n\star$ -continuous if $f^{-1}(G)$ is $n\star$ -closed in $(K, \mathcal{N}, \mathcal{I})$ for every n -closed set G of (L, \mathcal{N}') .

Definition 2.4. A map $f : (K, \mathcal{N}, \mathcal{I}) \rightarrow (L, \mathcal{N}', \mathcal{J})$ is said to be $n\mathcal{I}_g$ -continuous [2, 4] (resp. $n\mathcal{I}_{\tilde{g}}$ -continuous [5]) if $f^{-1}(G)$ is $n\mathcal{I}_g$ -closed (resp. $n\mathcal{I}_{\tilde{g}}$ -closed) in $(K, \mathcal{N}, \mathcal{I})$ for every n -closed set G of (L, \mathcal{N}') .

Definition 2.5. [5] A map $f : (O, \mathcal{N}, \mathcal{I}) \rightarrow (P, \mathcal{N}', \mathcal{J})$ is called $n\mathcal{I}_{\tilde{g}}$ -irresolute if $f^{-1}(G)$ is a $n\mathcal{I}_{\tilde{g}}$ -closed set of $(O, \mathcal{N}, \mathcal{I})$ for every $n\mathcal{I}_{\tilde{g}}$ -closed set G of $(P, \mathcal{N}', \mathcal{J})$.

Definition 2.6. [1] A map $f : (O, \mathcal{N}) \rightarrow (P, \mathcal{N}')$ is said to be n -closed map if for every n -closed subset G of (O, \mathcal{N}) , $f(G)$ is n -closed in (P, \mathcal{N}') .

Definition 2.7. [6] A map $f : (O, \mathcal{N}, \mathcal{I}) \rightarrow (P, \mathcal{N}', \mathcal{J})$ is called

- (1) $n\star$ -closed if for every n -closed subset G of $(O, \mathcal{N}, \mathcal{I})$, $f(G)$ is $n\star$ -closed in $(P, \mathcal{N}', \mathcal{J})$.
- (2) $n\mathcal{I}_g$ -closed (resp. $n\mathcal{I}_g$ -open) map if for every n -closed (resp. n -open) subset G of $(O, \mathcal{N}, \mathcal{I})$, $f(G)$ is $n\mathcal{I}_g$ -closed (resp. $n\mathcal{I}_g$ -open) in $(P, \mathcal{N}', \mathcal{J})$.

Definition 2.8. [6] A bijection $f : (O, \mathcal{N}, \mathcal{I}) \rightarrow (P, \mathcal{N}', \mathcal{J})$ is called an $n\star$ -Homeomorphism, if both f and f^{-1} are $n\star$ -continuous.

Definition 2.9. [6] A bijection $f : (O, \mathcal{N}, \mathcal{I}) \rightarrow (P, \mathcal{N}', \mathcal{J})$ is called an $n\mathcal{I}_g$ -Homeomorphism, if both f and f^{-1} are $n\mathcal{I}_g$ -continuous.

Theorem 2.1. [3]

- (1) Every $n\star$ -closed set is $n\mathcal{I}_{\tilde{g}}$ -closed but not conversely.
- (2) Every $n\mathcal{I}_{\tilde{g}}$ -closed set is $n\mathcal{I}_g$ -closed but not conversely.

Proposition 2.1. [5]

- (1) Every $n\star$ -continuous is $n\mathcal{I}_{\tilde{g}}$ -continuous but not conversely.
- (2) Every $n\mathcal{I}_{\tilde{g}}$ -continuous set is $n\mathcal{I}_g$ -continuous but not conversely.

3. $n\mathcal{I}_{\tilde{g}}$ -Closed and $n\mathcal{I}_{\tilde{g}}$ -Open maps

In this section, we introduce the concept of $n\mathcal{I}_{\tilde{g}}$ -closed maps and $n\mathcal{I}_{\tilde{g}}$ -open maps.

Definition 3.1. A map $f : (O, \mathcal{N}, \mathcal{I}) \rightarrow (P, \mathcal{N}', \mathcal{J})$ is said to be $n\mathcal{I}_{\tilde{g}}$ -closed (resp. $n\mathcal{I}_{\tilde{g}}$ -open) map if for every n -closed (resp. n -open) subset G of $(O, \mathcal{N}, \mathcal{I})$. $f(G)$ is $n\mathcal{I}_{\tilde{g}}$ -closed (resp. $n\mathcal{I}_{\tilde{g}}$ -open) in $(P, \mathcal{N}', \mathcal{J})$.

Theorem 3.1. A map $f : (O, \mathcal{N}, \mathcal{I}) \rightarrow (P, \mathcal{N}', \mathcal{J})$ is $n\mathcal{I}_{\tilde{g}}$ -closed if and only if for each n -open set U containing $f^{-1}(S)$, there is an $n\mathcal{I}_{\tilde{g}}$ -open set V of $(P, \mathcal{N}', \mathcal{J})$ such that $S \subseteq V$ and $f^{-1}(V) \subseteq U$.

Proof. Necessity: Let U be n -open in $(O, \mathcal{N}, \mathcal{I})$. Then U^c is n -closed in $(O, \mathcal{N}, \mathcal{I})$. Since f is an $n\mathcal{I}_{\tilde{g}}$ -closed map. $f(U^c)$ is $n\mathcal{I}_{\tilde{g}}$ -closed in $(P, \mathcal{N}', \mathcal{J})$. Thus $(P, \mathcal{N}', \mathcal{J}) - f(U^c)$ is $n\mathcal{I}_{\tilde{g}}$ -open, say V containing S such that $f^{-1}(V) \subseteq f^{-1}((P, \mathcal{N}', \mathcal{J}) - f(U^c)) = U$.

Sufficiency: Let F be a n -closed set in $(O, \mathcal{N}, \mathcal{I})$. Then F^c is n -open in $(O, \mathcal{N}, \mathcal{I})$. By hypothesis, there exists an $n\mathcal{I}_{\tilde{g}}$ -open set V of $(P, \mathcal{N}', \mathcal{J})$ such that $S \subseteq V$ and $f^{-1}(V) \subseteq F^c$ and so $F \subseteq (f^{-1}(V))^c = f^{-1}(V^c)$. Therefore, $f(F) = V^c$. Since V^c is $n\mathcal{I}_{\tilde{g}}$ -closed, $f(F)$ is $n\mathcal{I}_{\tilde{g}}$ -closed in $(P, \mathcal{N}', \mathcal{J})$. Hence f is $n\mathcal{I}_{\tilde{g}}$ -closed. \square

Theorem 3.2. If $f : (O, \mathcal{N}, \mathcal{I}) \rightarrow (P, \mathcal{N}', \mathcal{J})$ is a n -closed map, then it is $n\mathcal{I}_{\tilde{g}}$ -closed map.

Proof. Let G be a n -closed subset of $(O, \mathcal{N}, \mathcal{I})$. since f is a n -closed map, $f(G)$ is n -closed in $(P, \mathcal{N}', \mathcal{J})$. Every n -closed is $n\mathcal{I}_{\tilde{g}}$ -closed, $f(G)$ is $n\mathcal{I}_{\tilde{g}}$ -closed in $(P, \mathcal{N}', \mathcal{J})$. Hence f is $n\mathcal{I}_{\tilde{g}}$ -closed map. \square

Example 3.1. Let $O = \{5, 6, 7\}$, with $O/R = \{\{5\}, \{6, 7\}\}$ and $X = \{5, 6\}$. Then the Nano topology $\mathcal{N} = \{\phi, \{5\}, \{6, 7\}, O\}$ and $\mathcal{I} = \{\emptyset\}$. Let $P = \{5, 6, 7\}$, with $P/R = \{\{7\}, \{5, 6\}, \{6, 5\}\}$ and $X = \{5, 6\}$. Then the Nano topology $\mathcal{N}' = \{\phi, \{5, 6\}, P\}$ and $\mathcal{J} = \{\emptyset, \{5\}\}$. Then $n\mathcal{I}_{\tilde{g}}$ -closed sets are $\phi, P, \{5\}, \{7\}, \{5, 7\}, \{6, 7\}$. Define $f : (O, \mathcal{N}, \mathcal{I}) \rightarrow (P, \mathcal{N}', \mathcal{J})$ be the identity map. Then f is $n\mathcal{I}_{\tilde{g}}$ -closed map but not n -closed map because the subset $\{5\}$ is n -closed in $(O, \mathcal{N}, \mathcal{I})$, $f(\{5\}) = \{5\}$ is not n -closed in $(P, \mathcal{N}', \mathcal{J})$.

Theorem 3.3. If $f : (O, \mathcal{N}, \mathcal{I}) \rightarrow (P, \mathcal{N}', \mathcal{J})$ is a $n\star$ -closed map, then it is $n\mathcal{I}_{\tilde{g}}$ -closed map.

Proof. Let G be a n -closed subset of $(O, \mathcal{N}, \mathcal{I})$. since f is a $n\star$ -closed map, $f(G)$ is $n\star$ -closed in $(P, \mathcal{N}', \mathcal{J})$. Every $n\star$ -closed is $n\mathcal{I}_{\tilde{g}}$ -closed, $f(G)$ is $n\mathcal{I}_{\tilde{g}}$ -closed in $(P, \mathcal{N}', \mathcal{J})$. Hence f is $n\mathcal{I}_{\tilde{g}}$ -closed map. \square

Example 3.2. Let $O, \mathcal{N}, \mathcal{I}, P, \mathcal{N}', \mathcal{J}$ and f be defined as an Example 3.1. Then $n\star$ -closed sets are $\phi, P, \{5\}, \{7\}, \{5, 7\}$. Then f is $n\mathcal{I}_{\tilde{g}}$ -closed map but not $n\star$ -closed map because the subset $\{6, 7\}$ is n -closed in $(O, \mathcal{N}, \mathcal{I})$, $f(\{6, 7\}) = \{6, 7\}$ is not $n\star$ -closed in $(P, \mathcal{N}', \mathcal{J})$.

Theorem 3.4. If $f : (O, \mathcal{N}, \mathcal{I}) \rightarrow (P, \mathcal{N}', \mathcal{J})$ is a $n\mathcal{I}_{\tilde{g}}$ -closed map, then it is $n\mathcal{I}_g$ -closed map.

Proof. Let G be a n -closed subset of $(O, \mathcal{N}, \mathcal{I})$. since f is a $n\mathcal{I}_{\tilde{g}}$ -closed map, $f(G)$ is $n\mathcal{I}_{\tilde{g}}$ -closed in $(P, \mathcal{N}', \mathcal{J})$. Every $n\mathcal{I}_{\tilde{g}}$ -closed is $n\mathcal{I}_g$ -closed, $f(G)$ is $n\mathcal{I}_g$ -closed in $(P, \mathcal{N}', \mathcal{J})$. Hence f is $n\mathcal{I}_g$ -closed map. \square

Example 3.3. Let $O = \{5, 6, 7\}$, with $O/R = \{\{7\}, \{5, 6\}, \{6, 7\}\}$ and $X = \{5, 7\}$. Then the Nano topology $\mathcal{N} = \{\phi, \{7\}, \{5, 6\}, O\}$ and $\mathcal{I} = \{\emptyset, \{7\}\}$. Let $P = \{5, 6, 7\}$, with $P/R = \{\{5\}, \{6, 7\}\}$ and $X = \{5\}$. Then the Nano topology $\mathcal{N}' = \{\phi, \{5\}, P\}$ and $\mathcal{J} = \{\emptyset\}$. Then $\mathcal{I}_{\tilde{g}}$ -closed sets are $\phi, P, \{6, 7\}$ and $n\mathcal{I}_g$ -closed sets are $\phi, P, \{6\}, \{7\}, \{5, 6\}, \{5, 7\}, \{6, 7\}$. Then f is $n\mathcal{I}_g$ -closed map but not $n\mathcal{I}_{\tilde{g}}$ -closed map because the subset $\{5, 6\}$ is n -closed in $(O, \mathcal{N}, \mathcal{I})$, $f(\{5, 6\}) = \{5, 6\}$ is not $n\mathcal{I}_{\tilde{g}}$ -closed in $(P, \mathcal{N}', \mathcal{J})$.

4. $n\mathcal{I}_{\tilde{g}}$ -Homeomorphisms

Definition 4.1. A bijection $f : (O, \mathcal{N}, \mathcal{I}) \rightarrow (P, \mathcal{N}', \mathcal{J})$ is called an $n\mathcal{I}_{\tilde{g}}$ -Homeomorphism, if both f and f^{-1} are $n\mathcal{I}_{\tilde{g}}$ -continuous.

We say that the space $(O, \mathcal{N}, \mathcal{I})$ and $(P, \mathcal{N}', \mathcal{J})$ are $n\mathcal{I}_{\tilde{g}}$ -Homeomorphism if there exists an $n\mathcal{I}_{\tilde{g}}$ -Homeomorphic from $(O, \mathcal{N}, \mathcal{I})$ onto $(P, \mathcal{N}', \mathcal{J})$.

Theorem 4.1. Every $n\star$ -Homeomorphism is an $n\mathcal{I}_{\tilde{g}}$ -Homeomorphism.

Proof. Let $f : (O, \mathcal{N}, \mathcal{I}) \rightarrow (P, \mathcal{N}', \mathcal{J})$ is be a $n\star$ -homeomorphism. Then f and f^{-1} are $n\star$ -continuous and f is a bijection. since every $n\star$ -continuous map is $n\mathcal{I}_{\tilde{g}}$ -continuous, it follows that f is $n\mathcal{I}_{\tilde{g}}$ -Homeomorphism. \square

Example 4.1. Let $O = \{5, 6, 7, 8\}$, with $O/R = \{\{7\}, \{8\}, \{5, 6\}\}$ and $X = \{5\}$. Then the Nano topology $\mathcal{N} = \{\phi, \{5, 6\}, O\}$ and $\mathcal{I} = \{\emptyset, \{7\}\}$. Let $P = \{5, 6, 7, 8\}$, with $P/R = \{\{5\}, \{8\}, \{5, 6\}\}$ and $X = \{5, 6\}$. Then the Nano topology $\mathcal{N}' = \{\phi, \{5\}, \{6\}, \{5, 6\}, P\}$ and $\mathcal{J} = \{\emptyset, \{5\}\}$. Then $n\star$ -closed sets are $\phi, O, \{7\}, \{7, 8\}$, $n\mathcal{I}_{\tilde{g}}$ -closed sets are $\phi, O, \{7\}, \{7, 8\}, \{5, 7, 8\}, \{6, 7, 8\}$, $n\star$ -closed sets are $\phi, P, \{5\}, \{7, 8\}, \{5, 7, 8\}, \{6, 7, 8\}$ and $n\mathcal{I}_{\tilde{g}}$ -closed sets are $\phi, P, \{5\}, \{7, 8\}, \{5, 7, 8\}, \{6, 7, 8\}$. Define $f : (O, \mathcal{N}, \mathcal{I}) \rightarrow (P, \mathcal{N}', \mathcal{J})$ be the identity map. Then f is $n\mathcal{I}_{\tilde{g}}$ -homeomorphism but not $n\star$ -homeomorphism because the subset $f^{-1}(\{6, 7, 8\}) = \{6, 7, 8\}$ is n -closed in $(P, \mathcal{N}', \mathcal{J})$ but not $n\star$ -closed in $(O, \mathcal{N}, \mathcal{I})$.

Theorem 4.2. Every $n\mathcal{I}_{\tilde{g}}$ -Homeomorphism is an $n\mathcal{I}_g$ -Homeomorphism.

Proof. Let $f : (O, \mathcal{N}, \mathcal{I}) \rightarrow (P, \mathcal{N}', \mathcal{J})$ is be a $n\mathcal{I}_{\tilde{g}}$ -homeomorphism. Then f and f^{-1} are $n\mathcal{I}_{\tilde{g}}$ -continuous and f is a bijection. since every $n\mathcal{I}_{\tilde{g}}$ -continuous map is $n\mathcal{I}_g$ -continuous, it follows that f is $n\mathcal{I}_g$ -Homeomorphism. \square

Example 4.2. Let $O, \mathcal{N}, \mathcal{I}$ and f be defined as an Example 4.1. Let $P = \{5, 6, 7, 8\}$, with $P/R = \{\{7\}, \{8\}, \{5, 6\}\}$ and $X = \{5, 7\}$. Then the Nano topology $\mathcal{N}' = \{\phi, \{7\}, \{5, 6\}, \{5, 6, 7\}, P\}$ and $\mathcal{J} = \{\emptyset, \{5\}\}$. Then $n\mathcal{I}_g$ -closed sets are $\phi, O, \{7\}, \{8\}, \{5, 7\}, \{5, 8\}, \{6, 7\}, \{6, 8\}, \{7, 8\}, \{5, 6, 7\}, \{5, 6, 8\}, \{5, 7, 8\}, \{6, 7, 8\}$, $n\mathcal{I}_{\tilde{g}}$ -closed sets are $\phi, P, \{5\}, \{8\}, \{5, 8\}, \{7, 8\}, \{5, 6, 8\}, \{5, 7, 8\}$ and $n\mathcal{I}_g$ -closed sets are $\phi, P, \{5\}, \{8\}, \{5, 8\}, \{6, 8\}, \{7, 8\}, \{5, 6, 8\}, \{5, 7, 8\}, \{6, 7, 8\}$. Then f is $n\mathcal{I}_g$ -homeomorphism but not $n\mathcal{I}_{\tilde{g}}$ -homeomorphism because the subset $f^{-1}(\{5, 6, 8\}) = \{5, 6, 8\}$ is n -closed in $(P, \mathcal{N}', \mathcal{J})$ but not $n\mathcal{I}_{\tilde{g}}$ -closed in $(O, \mathcal{N}, \mathcal{I})$.

The following theorem gives a characterization of $n\mathcal{I}_{\tilde{g}}$ -homeomorphism.

Theorem 4.3. For any bijection $f : (O, \mathcal{N}, \mathcal{I}) \rightarrow (P, \mathcal{N}', \mathcal{J})$ the following statements are equivalent.

- (1) $f^{-1} : (P, \mathcal{N}', \mathcal{J}) \rightarrow (O, \mathcal{N}, \mathcal{I})$ is $n\mathcal{I}_{\tilde{g}}$ -continuous map.
- (2) f is an $n\mathcal{I}_{\tilde{g}}$ -open map.
- (3) f is an $n\mathcal{I}_{\tilde{g}}$ -closed map.

Proof. (1) \Rightarrow (2): Let G be an n -open set $(O, \mathcal{N}, \mathcal{I})$. By assumption $(f^{-1})^{-1}(G) = f(G)$ is $n\mathcal{I}_{\tilde{g}}$ -open in $(P, \mathcal{N}', \mathcal{J})$. so f is $n\mathcal{I}_{\tilde{g}}$ -open map.

(2) \Rightarrow (3): Let G be n -closed set of $(O, \mathcal{N}, \mathcal{I})$. Then G^c is n -open in $(O, \mathcal{N}, \mathcal{I})$. Since f is $n\mathcal{I}_{\tilde{g}}$ -open, $f(G^c)$ is $n\mathcal{I}_{\tilde{g}}$ -open in $(P, \mathcal{N}', \mathcal{J})$. This implies $[f(G)]^c$ is $n\mathcal{I}_{\tilde{g}}$ -open in $(P, \mathcal{N}', \mathcal{J})$. This implies $f(G)$ is $n\mathcal{I}_{\tilde{g}}$ -closed in $(P, \mathcal{N}', \mathcal{J})$. Therefore, f is an $n\mathcal{I}_{\tilde{g}}$ -closed map.

(3) \Rightarrow (1): Suppose G is n -closed set in $(O, \mathcal{N}, \mathcal{I})$. Then by assumption $(f^{-1})^{-1}(G) = f(G)$ is $n\mathcal{I}_{\tilde{g}}$ -closed in $(P, \mathcal{N}', \mathcal{J})$. Hence f^{-1} is $n\mathcal{I}_{\tilde{g}}$ -continuous map. \square

Theorem 4.4. Let $f : (O, \mathcal{N}, \mathcal{I}) \rightarrow (P, \mathcal{N}', \mathcal{J})$ be a bijective and $n\mathcal{I}_{\tilde{g}}$ -continuous map. Then the following statements are equivalent.

- (1) f is an $n\mathcal{I}_{\tilde{g}}$ -open map.
- (2) f is an $n\mathcal{I}_{\tilde{g}}$ -homeomorphism.
- (3) f is an $n\mathcal{I}_{\tilde{g}}$ -closed map.

Proof. Follows from Definitions 2.4, 3.1, 4.1 and Theorem 4.3. \square

5. $*n\mathcal{I}_{\tilde{g}}$ -Homeomorphisms

We introduce a new class of maps called $*n\mathcal{I}_{\tilde{g}}$ -Homeomorphisms which forms a subclass of $n\mathcal{I}_{\tilde{g}}$ -Homeomorphisms. This class of maps closed under composition of maps.

Definition 5.1. A map $f : (O, \mathcal{N}, \mathcal{I}) \rightarrow (P, \mathcal{N}', \mathcal{J})$ is called $*n\mathcal{I}_{\tilde{g}}$ -open if for every $n\mathcal{I}_{\tilde{g}}$ -open subset G of $(O, \mathcal{N}, \mathcal{I})$. $f(G)$ is $n\mathcal{I}_{\tilde{g}}$ -open in $(P, \mathcal{N}', \mathcal{J})$.

Theorem 5.1. For any bijection $f : (O, \mathcal{N}, \mathcal{I}) \rightarrow (P, \mathcal{N}', \mathcal{J})$ the following statements are equivalent.

- (1) The inverse map $f^{-1} : (P, \mathcal{N}', \mathcal{J}) \rightarrow (O, \mathcal{N}, \mathcal{I})$ is $n\mathcal{I}_{\tilde{g}}$ -irresolute.
- (2) f is an $*n\mathcal{I}_{\tilde{g}}$ -open map.

(3) f is an ${}^*n\mathcal{I}_{\tilde{g}}$ -closed map.

Proof. (1) \Rightarrow (2): Let G be $n\mathcal{I}_{\tilde{g}}$ -open in $(O, \mathcal{N}, \mathcal{I})$. By (1), $(f^{-1})^{-1}(G) = f(G)$ is $n\mathcal{I}_{\tilde{g}}$ -open in $(P, \mathcal{N}', \mathcal{J})$. Hence (2) holds.

(2) \Rightarrow (3): Let G be $n\mathcal{I}_{\tilde{g}}$ -closed in $(O, \mathcal{N}, \mathcal{I})$. Then $(O, \mathcal{N}, \mathcal{I}) - G$ is $n\mathcal{I}_{\tilde{g}}$ -open and by (2) $f((O, \mathcal{N}, \mathcal{I}) - G) = (P, \mathcal{N}', \mathcal{J}) - f(G)$ is $n\mathcal{I}_{\tilde{g}}$ -open in $(P, \mathcal{N}', \mathcal{J})$. That is $f(G)$ is $n\mathcal{I}_{\tilde{g}}$ -closed in $(P, \mathcal{N}', \mathcal{J})$ and so f is ${}^*n\mathcal{I}_{\tilde{g}}$ -closed map.

(3) \Rightarrow (1): Let G be $n\mathcal{I}_{\tilde{g}}$ -closed in $(O, \mathcal{N}, \mathcal{I})$. By (3), $f(G)$ is $n\mathcal{I}_{\tilde{g}}$ -closed in $(P, \mathcal{N}', \mathcal{J})$. But $f(G) = (f^{-1})^{-1}(G)$. Thus (1) holds. \square

Next we introduce a new class of maps as follows.

Definition 5.2. A bijection $f : (O, \mathcal{N}, \mathcal{I}) \rightarrow (P, \mathcal{N}', \mathcal{J})$ is called ${}^*n\mathcal{I}_{\tilde{g}}$ -Homeomorphisms if both f and f^{-1} are $n\mathcal{I}_{\tilde{g}}$ -irresolute.

We say that the spaces $(O, \mathcal{N}, \mathcal{I})$ and $(P, \mathcal{N}', \mathcal{J})$ are ${}^*n\mathcal{I}_{\tilde{g}}$ -Homeomorphic if there exists an ${}^*n\mathcal{I}_{\tilde{g}}$ -Homeomorphisms from $(O, \mathcal{N}, \mathcal{I})$ onto $(P, \mathcal{N}', \mathcal{J})$. The family of all $n\mathcal{I}_{\tilde{g}}$ -Homeomorphisms (resp ${}^*n\mathcal{I}_{\tilde{g}}$ -Homeomorphisms) from $(O, \mathcal{N}, \mathcal{I})$ onto $(P, \mathcal{N}', \mathcal{J})$ is denoted by $n\mathcal{I}_{\tilde{g}}\text{-h}(O, \mathcal{N}, \mathcal{I})$ (resp ${}^*n\mathcal{I}_{\tilde{g}}\text{-h}(O, \mathcal{N}, \mathcal{I})$).

Theorem 5.2. Every ${}^*n\mathcal{I}_{\tilde{g}}$ -Homeomorphism is an $n\mathcal{I}_{\tilde{g}}$ -Homeomorphism.

Proof. Let $f : (O, \mathcal{N}, \mathcal{I}) \rightarrow (P, \mathcal{N}', \mathcal{J})$ be a ${}^*n\mathcal{I}_{\tilde{g}}$ -homeomorphism. Then f and f^{-1} are $n\mathcal{I}_{\tilde{g}}$ irresolute and f is a bijection. Thus f and f^{-1} are $n\mathcal{I}_{\tilde{g}}$ -continuous. Hence f is $n\mathcal{I}_{\tilde{g}}$ -Homeomorphism. \square

Example 5.1. Let $O, \mathcal{N}, \mathcal{I}, P, \mathcal{N}', \mathcal{J}$ and f be defined as an Example 4.1. Then f is $n\mathcal{I}_{\tilde{g}}$ -homeomorphism but not ${}^*n\mathcal{I}_{\tilde{g}}$ -homeomorphism because the subset $\{5\}$ is $n\mathcal{I}_{\tilde{g}}$ -closed in $(P, \mathcal{N}', \mathcal{J})$, but $f^{-1} \{5\} = \{5\}$ is not $n\mathcal{I}_{\tilde{g}}$ -closed in $(O, \mathcal{N}, \mathcal{I})$.

Theorem 5.3. Let $f : (O, \mathcal{N}, \mathcal{I}) \rightarrow (P, \mathcal{N}', \mathcal{J})$ and $g : (P, \mathcal{N}', \mathcal{J}) \rightarrow (Q, \mathcal{N}'_*, \mathcal{K})$ be a ${}^*n\mathcal{I}_{\tilde{g}}$ -homeomorphism then their composition $g \circ f : (O, \mathcal{N}, \mathcal{I}) \rightarrow (Q, \mathcal{N}'_*, \mathcal{K})$ is ${}^*n\mathcal{I}_{\tilde{g}}$ -homeomorphism.

Proof. Let G be a $n\mathcal{I}_{\dot{g}}$ -open set in $(Q, \mathcal{N}'_*, \mathcal{K})$ since g is $n\mathcal{I}_{\dot{g}}$ -irresolute, $g^{-1}(G)$ is $n\mathcal{I}_{\dot{g}}$ -open set in $(P, \mathcal{N}', \mathcal{J})$. Since f is $n\mathcal{I}_{\dot{g}}$ -irresolute, $f^{-1}(g^{-1}(G)) = (g \circ f)^{-1}(G)$ is an $n\mathcal{I}_{\dot{g}}$ -open set in $(O, \mathcal{N}, \mathcal{I})$. Therefore $(O, \mathcal{N}, \mathcal{I})$ is $n\mathcal{I}_{\dot{g}}$ -irresolute. Also for a $n\mathcal{I}_{\dot{g}}$ -open set G in $(O, \mathcal{N}, \mathcal{I})$. We have $(g \circ f)(G) = g(f(G)) = g(S)$, where $S = f(G)$. By the hypothesis $f(G)$ is $n\mathcal{I}_{\dot{g}}$ -open in $(P, \mathcal{N}', \mathcal{J})$ and also by hypothesis $g(f(G))$ is a $n\mathcal{I}_{\dot{g}}$ -open in $(Q, \mathcal{N}'_*, \mathcal{K})$. That is $(g \circ f)(G)$ is a $n\mathcal{I}_{\dot{g}}$ -open set in $(Q, \mathcal{N}'_*, \mathcal{K})$ and therefore $(g \circ f)^{-1}$ is $n\mathcal{I}_{\dot{g}}$ -irresolute. Also $(g \circ f)$ is a bijection. Hence $(g \circ f)$ is $*n\mathcal{I}_{\dot{g}}$ -Homeomorphism. \square

Theorem 5.4. *The set $*n\mathcal{I}_{\dot{g}}\text{-h}(O, \mathcal{N}, \mathcal{I})$ is a group under the composition of maps.*

Proof. Define a binary operation $\star : *n\mathcal{I}_{\dot{g}}\text{-h}(O, \mathcal{N}, \mathcal{I}) \times *n\mathcal{I}_{\dot{g}}\text{-h}(O, \mathcal{N}, \mathcal{I})$ by $f \star g = (g \circ f)$ for all $f, g \in *n\mathcal{I}_{\dot{g}}\text{-h}(O, \mathcal{N}, \mathcal{I})$ and \circ is the usual operation of maps. Then by Theorem 5.3, $(g \circ f) \in *n\mathcal{I}_{\dot{g}}\text{-h}(O, \mathcal{N}, \mathcal{I})$. We know that composition of maps associative and the identity map $I : (O, \mathcal{N}, \mathcal{I}) \rightarrow (O, \mathcal{N}, \mathcal{I})$ belonging to $*n\mathcal{I}_{\dot{g}}\text{-h}(O, \mathcal{N}, \mathcal{I})$ serves as the identity element. for any $f \in *n\mathcal{I}_{\dot{g}}\text{-h}(O, \mathcal{N}, \mathcal{I})$, $f \circ f^{-1} = f^{-1} \circ f = I$. Hence inverse exists for each element of $*n\mathcal{I}_{\dot{g}}\text{-h}(O, \mathcal{N}, \mathcal{I})$. $*n\mathcal{I}_{\dot{g}}\text{-h}(O, \mathcal{N}, \mathcal{I})$ forms a group under the operation of composition of maps. \square

Theorem 5.5. *Let $f : (O, \mathcal{N}, \mathcal{I}) \rightarrow (P, \mathcal{N}', \mathcal{J})$ be an $*n\mathcal{I}_{\dot{g}}$ -homeomorphism. Then f induces an isomorphism from the group $*n\mathcal{I}_{\dot{g}}\text{-h}(O, \mathcal{N}, \mathcal{I})$ onto the group $*n\mathcal{I}_{\dot{g}}\text{-h}(P, \mathcal{N}', \mathcal{J})$.*

Proof. Let $f \in *n\mathcal{I}_{\dot{g}}\text{-h}(O, \mathcal{N}, \mathcal{I})$. We define a map $\phi_f : *n\mathcal{I}_{\dot{g}}\text{-h}(O, \mathcal{N}, \mathcal{I}) \rightarrow *n\mathcal{I}_{\dot{g}}\text{-h}(P, \mathcal{N}', \mathcal{J})$ by $\phi_f = f \circ h \circ f^{-1}$ for every $h \in *n\mathcal{I}_{\dot{g}}\text{-h}(O, \mathcal{N}, \mathcal{I})$. Then f is a bijection. Now for all $g, h \in *n\mathcal{I}_{\dot{g}}\text{-h}(O, \mathcal{N}, \mathcal{I})$, $\phi_f(g \circ h) = f \circ (g \circ h) \circ f^{-1} = (f \circ g \circ f^{-1}) \circ (f \circ h \circ f^{-1}) = \phi_f(g) \circ \phi_f(h)$. \square

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