Some Properties on Product of fs-Matrices and its Application in decision making

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***Abstract*— In this paper we introduced the commutative property to the products such as And product and Or product from the product of fs-matrices and also developed the complement for the complement of the products such as And, Or, And-Not, Or-Not products. In addition to that, found a new efficient solution procedure based on decision making problem.**

***Keywords*— Soft Set, Fuzzy Soft Matrices, Product of Fuzzy Soft Matrices, And-Not product, Decision Making**

I. INTRODUCTION

Soft set theory [12] was firstly introduced by Molodtsov in 1999 as a general mathematical tool for dealing with uncertain, fuzzy, not clearly defined objects.After presentation of the operations of soft sets [10], the properties and applications on the soft set theory have been studied increasingly [2,11]. The algebraic structure of soft set theory has also been studied in more detail [1,3]. In recent years, by embedding the ideas of fuzzy sets [17] many interesting applications of soft set theory have been done [5, 6]. Some recent works [13, 14,15,16] relates decision making problems with soft set theory.

To develop the soft set theory, operations of the soft sets are redefined to improve several new results and uni-int decision making method is constructed by using these new operations [8]. To make easy computation with the operations of soft sets,the soft matrix theory is presented and soft max-min decision making method is set up [9]. These decision making methods are more practical and can be successfully applied to many problems that contain uncertainties. In [6], a fuzzy soft (fs) set theory is defined. It allows constructing more efficient decision making method. Further, fs-matrices which are representation of the fs-sets. This representation has several advantages. It is easy to store and manipulate matrices and hence the fs-sets represented by them in a computer. Here, we also construct a fs-decision making method which is more practical and can be successfully applied to many problems. We finally give an example which shows that the method successfully works [7].

In this paper we introduced the commutative property to the products such as And product and Or product from the product of fs-matrices and also developed the complement for the complement of the products such as And, Or, And-Not, Or-Not products. In addition to that, found a new efficient solution procedure based on decision making problem.

II. PRELIMINARIES

Definition 1.1 [6] *Let U be an initial universe, E be the set of all parameters,$A \subseteq E$ and $\gamma_A(x)$ be a fuzzy set over U for all $x \in E$. Then, an fs-set Γ_A over U is a set defined by a function γ_A representing a mapping* $\gamma_A: E \rightarrow F(U)$ such that $\gamma_A(x) = \Phi; if\, x \notin A$

Here, γ_A is called fuzzy approximate function of the fs-set Γ_A, the value $\gamma_A(x)$ is a fuzzy set called x-element of the fs-set for all $x \in E$, and $\emptyset$ is the null fuzzy set. Thus, an fs-set Γ_A over U can be represented by the set of ordered pairs

$$\Gamma_A = \{(x, \gamma_A(x)): x \in E, \gamma_A(x) \in F(U)\}$$

Note that from now on, the sets of all fs-sets over U will be denoted by $FS(U)$.

Example 1.2 [7] Assume that $U = \{u_1, u_2, u_3, u_4, u_5\}$ is a universal set and $E = \{x_1, x_2, x_3, x_4\}$ is a set of all parameters. If $A = \{x_2, x_3, x_4\}$, $\gamma_A(x_2) = \{0.5/u_2, 0.8/u_4\}$, $\gamma_A(x_3) = \emptyset$ and $\gamma_A(x_4) = U$, then the fs-set Γ_A is written by $\Gamma_A = \{(x_2, \{0.5/u_2, 0.8/u_4\}), (x_4, U)\}$.

Definition 1.3 [7] *Let $\Gamma_A \in FS(U)$. Then a fuzzy relation form of Γ_A is defned by*

$$R_A = \{(\mu_{R_A}(u,x)/(u,x)) : (u,x) \in U \times E\},$$

where the membership function of μ_{R_A} is written by $\mu_{R_A}: U \times E \to [0,1], \mu_{R_A}(u,x) = \mu_{\gamma_{A(x)}}(u)$.

If $U = \{u_1, u_2, \ldots, u_m\}$, $E = \{x_1, x_2, \ldots x_n\}$ and $A \subseteq E$, then the R_A can be presented by a table as in the following form

R_A	x_1	x_2	$\cdots$	x_n
u_1	$\mu_{R_A}(u_1,x_1)$	$\mu_{R_A}(u_1,x_2)$	$\cdots$	$\mu_{R_A}(u_1,x_n)$
u_2	$\mu_{R_A}(u_2,x_1)$	$\mu_{R_A}(u_2,x_2)$	$\cdots$	$\mu_{R_A}(u_2,x_n)$
$\vdots$	$\vdots$	$\ddots$	$\vdots$	
u_m	$\mu_{R_A}(u_m,x_1)$	$\mu_{R_A}(u_m,x_2)$	$\cdots$	$\mu_{R_A}(u_m,x_n)$

If $a_{ij} = \mu_{R_A}(u_i, x_j)$, we can define a matrix

$$[\mathbf{a_{ij}}]_{\mathbf{m \times n}} = \begin{bmatrix} x_{11} & x_{12} & \cdots & x_{1n} \\ x_{21} & x_{22} & \cdots & x_{2n} \\ \vdots & \vdots & \ddots & \vdots \\ x_{m1} & x_{m2} & \cdots & x_{mn} \end{bmatrix}$$

which is called an $m \times n$ fs-matrix of the fs-set Γ_A over U.

According to this definition, an fs-set Γ_A is uniquely characterized by the matrix $[a_{ij}]_{m\times n}$. It means that an fs-set Γ_A is formally equal to its soft matrix $[a_{ij}]_{m\times n}$. Therefore, we shall identify any fs-set with its fs-matrix and use these two concepts as interchangeable.

The set of all $m \times n$ fs-matrices over U will be denoted by $FSM_{m\times n}$. From now on we shall delete the subscript $m \times n$ of $[a_{ij}]_{m\times n}$, we use $[a_{ij}]$ instead of $[a_{ij}]_{m\times n}$, since $[a_{ij}] \in FSM_{m\times n}$ means that $[a_{ij}]$ is an $m \times n$ fs-matrix for $i = 1,2,\ldots,m$ and $j = 1,2,\ldots,n$.

Example 1.4 [7] Let us consider Example 0.2 . Then the relation form of Γ_A is written by

$R_A = \{0.5/(u_2,x_2), 0.8/(u_4,x_2), 1/(u_1,x_4), 1/(u_2,x_4), 1/(u_3,x_4), 1/(u_4,x_4), 1/(u_5,x_4)\}$

Hence, the fs-matrix $[a_{ij}]$ is written by

$$[\mathbf{a_{ij}}] = \begin{bmatrix} 0 & 0 & 0 & 1 \\ 0 & 0.5 & 0 & 1 \\ 0 & 0 & 0 & 1 \\ 0 & 0.8 & 0 & 1 \\ 0 & 0 & 0 & 1 \end{bmatrix}$$

Definition 1.5 [7] *Let $[a_{ij}] \in FSM_{m\times n}$. Then $[a_{ij}]$ is called*

(1) a zero fs-matrix, denoted by [0], if $a_{ij} = 0$ for all i and j.

(2) an A-universal fs-matrix, denoted by $[\tilde{a}_{ij}]$, if $a_{ij} = 1$ for all $j \in I_A = \{j : x_j \in A\}$ and i.

(3) a universal fs-matrix, denoted by [1], if $a_{ij} = 1$ for all i and j.

Example 1.6 [7] Assume that $U = \{u_1, u_2, u_3, u_4, u_5\}$ is a universal set, $E = \{x_1, x_2, x_3, x_4\}$ is a set of parameters, $A \subseteq E$, $\gamma_A(x)$ is a fuzzy set over U for all $x \in E$ and $[a_{ij}], [b_{ij}], [c_{ij}] \in FSM_{5\times 4}$.

If $A = \{x_1, x_3\}$ and $\gamma_A(x_1) = \emptyset$, $\gamma_A(x_3) = \emptyset$, then $[a_{ij}] = [0]$ is a zero fs-matrix written by

$$[\mathbf{0}] = \begin{bmatrix} 0 & 0 & 0 & 0 \\ 0 & 0 & 0 & 0 \\ 0 & 0 & 0 & 0 \\ 0 & 0 & 0 & 0 \\ 0 & 0 & 0 & 0 \end{bmatrix}$$

If $B = \{x_1, x_2\}$ and $\gamma_B(x_1) = U$, $\gamma_B(x_2) = U$, then $[b_{ij}] = [\tilde{b}_{ij}]$ is a B-universal fs-matrix written by

$$[\tilde{\mathbf{b}}_{\mathbf{ij}}] = \begin{bmatrix} 1 & 1 & 0 & 0 \\ 1 & 1 & 0 & 0 \\ 1 & 1 & 0 & 0 \\ 1 & 1 & 0 & 0 \\ 1 & 1 & 0 & 0 \end{bmatrix}$$

If $C = E$, and $\gamma_C(x_i) = U$ for all $x_i \in C$, $i = 1,2,3,4$, then $[c_{ij}] = [1]$ is a universal fs-matrix written by

$$[\mathbf{1}] = \begin{bmatrix} 1 & 1 & 1 & 1 \\ 1 & 1 & 1 & 1 \\ 1 & 1 & 1 & 1 \\ 1 & 1 & 1 & 1 \\ 1 & 1 & 1 & 1 \end{bmatrix}$$

Definition 1.7 [7] *Let* $[a_{ij}], [b_{ij}] \in FSM_{m\times n}$. *Then*

(1) $[a_{ij}]$ is a fs-submatrix of $[b_{ij}]$, denoted by $[a_{ij}] \tilde{\subseteq} [bij]$, if $a_{ij} \le b_{ij}$ for all i and j.

(2) $[a_{ij}]$ is a proper fs-submatrix of $[b_{ij}]$, denoted by $[a_{ij}] \tilde{\subset} [b_{ij}]$, if $a_{ij} \le b_{ij}$ for all i and j and for at least one term $a_{ij} < b_{ij}$.

(3) $[a_{ij}]$ and $[b_{ij}]$ are fs-equal matrices, denoted by $[a_{ij}] = [b_{ij}]$, if $a_{ij} = b_{ij}$ for all i and j.

Definition 1.8 [7] *Let* $[a_{ij}], [b_{ij}] \in FSM_{m\times n}$. *Then the fs-matrix* $[c_{ij}]$ *is called*

(1) union of $[a_{ij}]$ and $[b_{ij}]$, denoted $[a_{ij}] \tilde{\cup} [b_{ij}]$, if $c_{ij} = max\{a_{ij}, b_{ij}\}$ for all i and j.

(2) intersection of $[a_{ij}]$ and $[b_{ij}]$, denoted $[a_{ij}] \tilde{\cap} [b_{ij}]$, if $c_{ij} = min\{a_{ij}, b_{ij}\}$ for all i and j.

(3) complement of $[a_{ij}]$, denoted by $[a_{ij}]^o$, if $c_{ij} = 1 - a_{ij}$ for all i and j.

Definition 1.9 [7] *Let* $[a_{ij}], [b_{ij}] \in FSM_{m\times n}$. *Then* $[a_{ij}]$ *and* $[b_{ij}]$ *are disjoint, if* $[a_{ij}] \tilde{\cap} [b_{ij}] = [0]$ *for all i and j.*

Example 1.10 [7] Assume that

$$[\mathbf{a}_{\mathbf{ij}}] = \begin{bmatrix} 0 & 0.6 & 0 & 0 \\ 0.1 & 0 & 1 & 0 \\ 0 & 0.3 & 0.8 & 0 \\ 0.7 & 0 & 0 & 0 \\ 0 & 1 & 0 & 0 \end{bmatrix} \quad [\mathbf{b}_{\mathbf{ik}}] = \begin{bmatrix} 0 & 0 & 0.7 & 0.4 \\ 0 & 0.2 & 0 & 1 \\ 0 & 0 & 0.5 & 0.9 \\ 0 & 0.8 & 0 & 1 \\ 0 & 0 & 0 & 0.3 \end{bmatrix}$$

Then, $[a_{ij}] \tilde{\cap} [b_{ij}] = [0]$ and

$$[\mathbf{a}_{\mathbf{ij}}] \tilde{\cup} [\mathbf{b}_{\mathbf{ij}}] = \begin{bmatrix} 0 & 0.6 & 0.7 & 0.4 \\ 0.1 & 0.2 & 1 & 1 \\ 0 & 0.3 & 0.8 & 0.9 \\ 0.7 & 0 & 0.5 & 1 \\ 0 & 1 & 0 & 0.3 \end{bmatrix} \quad [\mathbf{a}_{\mathbf{ij}}]^{\mathbf{o}} = \begin{bmatrix} 1 & 0.4 & 1 & 1 \\ 0.9 & 1 & 0 & 1 \\ 1 & 0.7 & 0.2 & 1 \\ 0.3 & 1 & 1 & 1 \\ 1 & 0 & 1 & 1 \end{bmatrix}$$

Definition 1.11 [7] *Let* $[a_{ij}], [b_{ik}] \in FSM_{m\times n}$. *Then And-product of* $[a_{ij}]$ *and* $[b_{ik}]$ *is defined by*

$\wedge: FSM_{m\times n} \times FSM_{m\times n} \to FSM_{m\times n^2}, [a_{ij}] \wedge [b_{ik}] = [c_{ip}]$

where $c_{ip} = min\{a_{ij}, b_{ik}\}$ such that $p = n(j-1) + k$.

Definition 1.12 [7] *Let* $[a_{ij}], [b_{ik}] \in FSM_{m\times n}$. *Then Or-product of* $[a_{ij}]$ *and* $[b_{ik}]$ *is defined by*

$\vee: FSM_{m\times n} \times FSM_{m\times n} \to FSM_{m\times n^2}, [a_{ij}] \vee [b_{ik}] = [c_{ip}]$

where $c_{ip} = max\{a_{ij}, b_{ik}\}$ such that $p = n(j-1) + k$.

Definition 1.13 [7] *Let* $[a_{ij}], [b_{ik}] \in FSM_{m\times n}$. *Then And-Not-product of* $[a_{ij}]$ *and* $[b_{ik}]$ *is defined by*

$\overline{\wedge}: FSM_{m\times n} \times FSM_{m\times n} \to FSM_{m\times n^2}, [a_{ij}] \overline{\wedge} [b_{ik}] = [c_{ip}]$

where $c_{ip} = min\{a_{ij}, 1 - b_{ik}\}$ such that $p = n(j-1) + k$.

Definition 1.14 [7] *Let* $[a_{ij}],[b_{ik}] \in FSM_{m\times n}$. *Then Or-Not-product of* $[a_{ij}]$ *and* $[b_{ik}]$ *is defined by*
$\underline{\vee}: FSM_{m\times n} \times FSM_{m\times n} \to FSM_{m\times n^2}, [a_{ij}] \underline{\vee} [b_{ik}] = [c_{ip}]$
where $c_{ip} = max\{a_{ij}, 1 - b_{ik}\}$ such that $p = n(j-1)+k$.

Example 1.15 [7] Assume that $[a_{ij}],[b_{ik}] \in FSM_{2\times 3}$ are given as follows

$$[\mathbf{a_{ij}}] = \begin{bmatrix} 0 & 0 & 0.3 \\ 0 & 1 & 0.7 \end{bmatrix} \quad [\mathbf{b_{ik}}] = \begin{bmatrix} 0.5 & 0 & 0.2 \\ 0.2 & 0 & 0 \end{bmatrix}$$

To calculate $[a_{ij}] \wedge [b_{ik}] = [c_{ip}]$, we have to find c_{ip} for all $i = 1,2$ and $p = 1,2,\ldots 9$. Let us find c_{17}. Since $n = 3$, $i = 1$ and $p = 7$, we get $j = 3$ and $k = 1$ from $7 = 3(j-1)+k$. Hence $c_{17} = min\{a_{13}, b_{11}\} = min\{0.3, 0.5\} = 0.3$. If the other entries of $[c_{ip}]$ can be found similarly, then we can obtain the matrix as follows;

$$[\mathbf{a_{ij}}] \wedge [\mathbf{b_{ij}}] = \begin{bmatrix} 0 & 0 & 0 & 0 & 0 & 0 & 0.3 & 0 & 0.2 \\ 0 & 0 & 0 & 0.2 & 0 & 0 & 0.2 & 0 & 0 \end{bmatrix}$$

Similarly, we can also find products $[a_{ij}] \vee [bik]$, $[a_{ij}] \underline{\vee} [b_{ik}]$ and $[a_{ij}] \overline{\wedge} [b_{ik}]$. Note that the commutativity is not valid for the products of fs-matrices.

Definition 1.16 [7] *Let* $[c_{ip}] \in FSMm \times n^2$, $I_k = \{p: \exists i; c_{ip} \neq 0;\ (k-1)n < p \leq kn\}$ *for all* $k \in I = \{1,2,\ldots\ldots,n\}$. *Then fs-max-min decision function, denoted* Mm*, is defined as follows*

$$Mm: FSM_{m\times n^2} \to FSM_{m\times 1}, \quad Mm[c_{ip}] = [d_{i1}] = [\max_k \{d_{ik}\}]$$

where

$$t_{ik} = \begin{cases} \min\limits_{p\in I_k}\{c_{ip}\} & \text{if} I_k \neq K \\ 0 & \text{if} I_k = \emptyset \end{cases}$$

The one column fs-matrix $Mm[c_{ip}]$ is called max-min decision fs-matrix.

Definition 1.17 [7] *Let* $U = \{u_1, u_2, \ldots., u_m\}$ *be an initial universe and* $Mm[c_{ip}] = [d_{i1}]$. *Then a subset of* U *can be obtained by using* $[d_{i1}]$ *as in the following way*

$$opt_{[d_{i1}]}(U) = \{d_{i1}/u_i : u_i \in U, d_{i1} \neq 0\}$$

which is called an optimum fuzzy set on U.

III. Some Properties on Product of FS-Matrices

Proposition 2.1
(1) $[a_{ij}] \wedge [b_{ik}] = [b_{ik}] \wedge [a_{ij}]$
(2) $[a_{ij}] \vee [b_{ik}] = [b_{ik}] \vee [a_{ij}]$
Proof:

(1) Assume that $[a_{ij}], [b_{ik}] \in FSM_{m\times n}$ are given as follows

$$[\mathbf{a_{ij}}]_{\mathbf{m\times n}} = \begin{bmatrix} a_{11} & a_{12} & \cdots & a_{1n} \\ a_{21} & a_{22} & \cdots & a_{2n} \\ \vdots & \vdots & \ddots & \vdots \\ a_{m1} & a_{m2} & \cdots & a_{mn} \end{bmatrix} \qquad [\mathbf{b_{ik}}]_{\mathbf{m\times n}} = \begin{bmatrix} b_{11} & b_{12} & \cdots & b_{1n} \\ b_{21} & b_{22} & \cdots & b_{2n} \\ \vdots & \vdots & \ddots & \vdots \\ b_{m1} & b_{m2} & \cdots & b_{mn} \end{bmatrix}$$

To calculate $[a_{ij}] \wedge [b_{ik}] = [c_{ip}]$, we have to find c_{ip} for all $i = 1, 2, ..., m$, where $c_{ip} = \min\{a_{ij}, b_{ik}\}$ such that $p = n(j-1) + k$. If the other entries of $[c_{ip}]$ can be found similarly, then we can obtain the matrix for $[a_{ij}] \wedge [b_{ik}]$.

Next to calculate $[b_{ik}] \wedge [a_{ij}] = [c_{ip}]$, we have to find c_{ip} for all $i = 1, 2, ..., m$, where $c_{ip} = \min\{b_{ik}, a_{ij}\}$ such that $p = n(j-1) + k$. If the other entries of $[c_{ip}]$ can be found similarly, then we can obtain the matrix for $[b_{ik}] \wedge [a_{ij}]$.

Hence $[a_{ij}] \wedge [b_{ik}] = [b_{ik}] \wedge [a_{ij}]$.

(2) Assume that $[a_{ij}], [b_{ik}] \in FSM_{m\times n}$ are given as follows

$$[\mathbf{a_{ij}}]_{\mathbf{m\times n}} = \begin{bmatrix} a_{11} & a_{12} & \cdots & a_{1n} \\ a_{21} & a_{22} & \cdots & a_{2n} \\ \vdots & \vdots & \ddots & \vdots \\ a_{m1} & a_{m2} & \cdots & a_{mn} \end{bmatrix} \qquad [\mathbf{b_{ik}}]_{\mathbf{m\times n}} = \begin{bmatrix} b_{11} & b_{12} & \cdots & b_{1n} \\ b_{21} & b_{22} & \cdots & b_{2n} \\ \vdots & \vdots & \ddots & \vdots \\ b_{m1} & b_{m2} & \cdots & b_{mn} \end{bmatrix}$$

To calculate $[a_{ij}] \vee [b_{ik}] = [c_{ip}]$, we have to find c_{ip} for all $i = 1, 2, ..., m$, where $c_{ip} = \max\{a_{ij}, b_{ik}\}$ such that $p = n(j-1) + k$. If the other entries of $[c_{ip}]$ can be found similarly, then we can obtain the matrix for $[a_{ij}] \vee [b_{ik}]$.

Next to calculate $[b_{ik}] \vee [a_{ij}] = [c_{ip}]$, we have to find c_{ip} for all $i = 1, 2, ..., m$, where $c_{ip} = \max\{b_{ik}, a_{ij}\}$ such that $p = n(j-1) + k$. If the other entries of $[c_{ip}]$ can be found similarly, then we can obtain the matrix for $[b_{ik}] \vee [a_{ij}]$.

Hence $[a_{ij}] \vee [b_{ik}] = [b_{ik}] \vee [a_{ij}]$.

Example 2.72 (1) Assume that $[a_{ij}], [b_{ik}] \in FSM_{2\times 3}$ are given as follows

$$[\mathbf{a_{ij}}] = \begin{bmatrix} 0.5 & 0 & 0.7 \\ 0.3 & 0 & 0.4 \end{bmatrix} \qquad [\mathbf{b_{ik}}] = \begin{bmatrix} 0.1 & 0.8 & 0 \\ 0 & 0.1 & 0 \end{bmatrix}$$

To calculate $[a_{ij}] \wedge [b_{ik}] = [c_{ip}]$, we have to find c_{ip} for all $i = 1,2$ and $p = 1,2,\ldots 9$. Let us find c_{29}. Since $n = 3$, $i = 2$ and $p = 9$, we get $j = 3$ and $k = 3$ from $9 = 3(j-1) + k$. Hence $c_{29} = min\{a_{23}, b_{23}\} = min\{0.4, 0\} = 0$. If the other entries of $[c_{ip}]$ can be found similarly, then we can obtain the matrix as follows;

$$[\mathbf{a_{ij}}] \wedge [\mathbf{b_{ik}}] = \begin{bmatrix} 0.1 & 0.5 & 0 & 0 & 0 & 0 & 0.1 & 0.7 & 0 \\ 0 & 0.1 & 0 & 0.2 & 0 & 0 & 0 & 0.1 & 0 \end{bmatrix}$$

Next to calculate $[b_{ik}] \wedge [a_{ij}] = [c_{ip}]$, we have to find c_{ip} for all $i = 1,2$ and $p = 1,2,\ldots 9$. Let us find c_{27}. Since $n = 3$, $i = 2$ and $p = 7$, we get $j = 3$ and $k = 1$ from $7 = 3(j-1) + k$. Hence $c_{27} = min\{b_{21}, a_{23}\} = min\{0,0\} = 0$. If the other entries of $[c_{ip}]$ can be found similarly, then we can obtain the matrix as follows;

$$[\mathbf{b_{ik}}] \wedge [\mathbf{a_{ij}}] = \begin{bmatrix} 0.1 & 0.5 & 0 & 0 & 0 & 0 & 0.1 & 0.7 & 0 \\ 0 & 0.1 & 0 & 0.2 & 0 & 0 & 0 & 0.1 & 0 \end{bmatrix}$$

Hence $[a_{ij}] \wedge [b_{ik}] = [b_{ik}] \wedge [a_{ij}]$

(2) Assume that $[a_{ij}], [b_{ik}] \in FSM_{2\times3}$ are given as follows

$$[\mathbf{a_{ij}}] = \begin{bmatrix} 0.5 & 0 & 0.7 \\ 0.3 & 0 & 0.4 \end{bmatrix} \quad [\mathbf{b_{ik}}] = \begin{bmatrix} 0.1 & 0.8 & 0 \\ 0 & 0.1 & 0 \end{bmatrix}$$

To calculate $[a_{ij}] \vee [b_{ik}] = [c_{ip}]$, we have to find c_{ip} for all $i = 1,2$ and $p = 1,2,\ldots 9$. Let us find c_{29}. Since $n = 3$, $i = 2$ and $p = 9$, we get $j = 3$ and $k = 3$ from $9 = 3(j-1)+k$. Hence $c_{29} = max\{a_{23}, b_{23}\} = max\{0.4,0\} = 0.4$. If the other entries of $[c_{ip}]$ can be found similarly, then we can obtain the matrix as follows;

$$[\mathbf{a_{ij}}] \vee [\mathbf{b_{ik}}] = \begin{bmatrix} 0.5 & 0.8 & 0.5 & 0.1 & 0.8 & 0 & 0.7 & 0.8 & 0.7 \\ 0.3 & 0.3 & 0.3 & 0 & 0.1 & 0 & 0.4 & 0.4 & 0.4 \end{bmatrix}$$

Next to calculate $[b_{ik}] \vee [a_{ij}] = [c_{ip}]$, we have to find c_{ip} for all $i = 1,2$ and $p = 1,2,\ldots 9$. Let us find c_{27}. Since $n = 3$, $i = 2$ and $p = 7$, we get $j = 3$ and $k = 1$ from $7 = 3(j-1)+k$. Hence $c_{27} = max\{b_{21}, a_{23}\} = min\{0,0.4\} = 0.4$. If the other entries of $[c_{ip}]$ can be found similarly, then we can obtain the matrix as follows;

$$[\mathbf{b_{ik}}] \vee [\mathbf{a_{ij}}] = \begin{bmatrix} 0.5 & 0.8 & 0.5 & 0.1 & 0.8 & 0 & 0.7 & 0.8 & 0.7 \\ 0.3 & 0.3 & 0.3 & 0 & 0.1 & 0 & 0.4 & 0.4 & 0.4 \end{bmatrix}$$

Hence $[a_{ij}] \vee [b_{ik}] = [b_{ik}] \vee [a_{ij}]$

Proposition 2.8.

(1) $[([a_{ij}] \wedge [b_{ik}])^o]^o = [a_{ij}] \wedge [b_{ik}]$

(2) $[([a_{ij}] \vee [b_{ik}])^o]^o = [a_{ij}] \vee [b_{ik}]$

(3) $[([a_{ij}] \overline{\wedge} [b_{ik}])^o]^o = [a_{ij}] \overline{\wedge} [b_{ik}]$

(4) $[([a_{ij}] \underline{\vee} [b_{ik}])^o]^o = [a_{ij}] \underline{\vee} [b_{ik}]$

Proof:

(1) Assume that $[a_{ij}]$, $[b_{ik}] \in FSM_{m\times n}$ are given as follows

$$[\mathbf{a_{ij}}]_{\mathbf{m\times n}} = \begin{bmatrix} a_{11} & a_{12} & \cdots & a_{1n} \\ a_{21} & a_{22} & \cdots & a_{2n} \\ \vdots & \vdots & \ddots & \vdots \\ a_{m1} & a_{m2} & \cdots & a_{mn} \end{bmatrix} \quad [\mathbf{b_{ik}}]_{\mathbf{m\times n}} = \begin{bmatrix} b_{11} & b_{12} & \cdots & b_{1n} \\ b_{21} & b_{22} & \cdots & b_{2n} \\ \vdots & \vdots & \ddots & \vdots \\ b_{m1} & b_{m2} & \cdots & b_{mn} \end{bmatrix}$$

Let $[a_{ij}] \wedge [b_{ik}] = [c_{ip}]$, where $c_{ip} = \min\{a_{ij}, b_{ik}\}$. Then,

$([a_{ij}] \wedge [b_{ik}])^o = [c_{ip}]^o$ and $(([a_{ij}] \wedge [b_{ik}])^o)^o = ([c_{ip}]^o)^o = [c_{ip}] = [a_{ij}] \wedge [b_{ik}]$.

Hence $(([a_{ij}] \wedge [b_{ik}])^o)^o = ([c_{ip}]^o)^o = [c_{ip}] = [a_{ij}] \wedge [b_{ik}]$.

Similarly, we get (2), (3), (4).

Example 2.9

Assume that $[a_{ij}], [b_{ik}] \in FSM_{2\times3}$ are given as follows:

$$[\mathbf{a_{ij}}] = \begin{bmatrix} 0.5 & 0 & 0.7 \\ 0.3 & 0 & 0.4 \end{bmatrix} \quad [\mathbf{b_{ik}}] = \begin{bmatrix} 0.1 & 0.8 & 0 \\ 0 & 0.1 & 0 \end{bmatrix}$$

Then,

(1) $$[\mathbf{a_{ij}}] \wedge [\mathbf{b_{ik}}] = \begin{bmatrix} 0.1 & 0.5 & 0 & 0 & 0 & 0 & 0.1 & 0.7 & 0 \\ 0 & 0.1 & 0 & 0.2 & 0 & 0 & 0 & 0.1 & 0 \end{bmatrix}$$

$$[([\mathbf{a_{ij}}] \wedge [\mathbf{b_{ik}}])^{\mathbf{o}}]^{\mathbf{o}} = \begin{bmatrix} 0.1 & 0.5 & 0 & 0 & 0 & 0 & 0.1 & 0.7 & 0 \\ 0 & 0.1 & 0 & 0.2 & 0 & 0 & 0 & 0.1 & 0 \end{bmatrix}$$

Hence $[([a_{ij}] \wedge [b_{ik}])^o]^o = [a_{ij}] \wedge [b_{ij}]$.

(2) Similarly, we have
$[([a_{ij}] \vee [b_{ik}])^o]^o = [a_{ij}] \vee [b_{ik}]$

(3) $[\mathbf{a_{ij}}] \overline{\wedge} [\mathbf{b_{ik}}] = \begin{bmatrix} 0.5 & 0.2 & 0.5 & 0 & 0 & 0 & 0.7 & 0.2 & 0.7 \\ 0.3 & 0.3 & 0.3 & 0 & 0 & 0 & 0.4 & 0.4 & 0.4 \end{bmatrix}$

$$[([\mathbf{a_{ij}}] \overline{\wedge} [\mathbf{b_{ik}}])^{\mathbf{o}}]^{\mathbf{o}} = \begin{bmatrix} 0.5 & 0.2 & 0.5 & 0 & 0 & 0 & 0.7 & 0.2 & 0.7 \\ 0.3 & 0.3 & 0.3 & 0 & 0 & 0 & 0.4 & 0.4 & 0.4 \end{bmatrix}$$

Hence $[([a_{ij}] \overline{\wedge} [b_{ik}])^o]^o = [a_{ij}] \overline{\wedge} [b_{ik}]$.

(4) Similarly, we have $[([a_{ij}] \underline{\vee} [b_{ik}])^o]^o = [a_{ij}] \underline{\vee} [b_{ik}]$.

3 Application

In this section, we are going to use fs-max-min decision making (FSMmDM) method [7] by using fs-max-min decision function. In [7] the decision making is done by two decision makers. In this paper, more decision makers are taken into consideration.

Assume that a company manager has a set of different types of areas U = {Velachery, Ambattur, Porur} which may be characterized by a set of parameters

E = {Transfort specilities, Eco friendly, Source materials availability, Residential area}.

Suppose a company's managers such as Sales Manager, Project Manager, Production Manager plans to construct a office building. Each manager has to consider their own set of parameters, on considering those parameters they have to choose a location with suitable aspects to construct a office building by using the FSMmDM as follows.

Assume that U = {Velachery, Ambattur, Porur} is a universal set and E = {Transfort specilities, Eco friendly, Source materials availability, Resitancial area} is a set of all parameters.

Step 1: First, Sales Manager, Project Manager, Production Manager have to choose the sets of their parameters, A = {Transfort specilities, Eco friendly, Resitancial area} and B = {Eco friendly, source materials availability}, C= {Transfort specilities, Eco friendly, Source materials availability, Resitancial area} respectively.

Step 2:
Then we can write the following fs-matrices which are constructed according to their parameters.

$$[\mathbf{a_{ij}}] = \begin{bmatrix} 0.7 & 0.4 & 0 \\ 1 & 0 & 0 \\ 0 & 0 & 0 \end{bmatrix} \quad [\mathbf{b_{ik}}] = \begin{bmatrix} 0 & 0.4 & 0.9 \\ 0 & 0.7 & 0.3 \\ 0 & 0 & 0 \end{bmatrix} \quad [\mathbf{f_{il}}] = \begin{bmatrix} 0.1 & 0.3 & 0.4 \\ 0.2 & 0 & 0.5 \\ 0 & 0 & 0 \end{bmatrix}$$

Step 3: Now, we can find a product of the fs-matrices $[a_{ij}]$, $[b_{ik}]$ and $[\mathbf{f_{il}}]$ Type equation here.by using And-product as follows

$$\begin{bmatrix} 0 & 0 & 0 & 0.1 & 0.3 & 0.4 & 0.1 & 0.3 & 0.4 & 0 & 0 & 0 & 0.1 & 0.3 & 0.4 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 \\ 0 & 0 & 0 & 0.2 & 0 & 0.5 & 0.2 & 0 & 0.3 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 \\ 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 \end{bmatrix}$$

Here, we use And-product since Sales Manager,Project Manager and Production Manager choices have to be considered.

Step 4:

To calculate $Mm(([a_{ij}] \wedge [b_{ik}]) \wedge [f_{il}]) = [d_{i1}]$, we have to find d_{i1} for all $i \in \{1,2,3\}$. To demonstrate, let us find d_{11}, d_{21}, d_{31}. Since $i = 1$ and , $k \in \{1,2,3\}$,

$$d_{11} = \max_k\{t_{1k}\} = \max\{t_{11}, t_{12}, t_{13}\}$$

Here, we have to find t_{1k}, t_{2k}, t_{3k} for all $k \in \{1,2,3\}$. To demonstrate, let us find t_{11} , t_{32} , t_{3k} . $I_1 = \{p: c_{ip} \neq 0, 0 < p \leq 9\} = \{1,2,3,4,5,6,7,8,9\}$; for $k = 1$ and $n = 9$ and $I_2 = \{p: c_{ip} \neq 0, 9 < p \leq 18\} = \{10,11,12,13,14,15,16,17,18\}$ for $k = 2$ and $n = 9$ and $I_3 = \{p: c_{ip} \neq 0, 18 < p \leq 27\} = \emptyset$ for $k = 3$ and $n = 9$. Hence $t_{11} = 0.1$ and $t_{12} = \min\{c_{10}, c_{11}, c_{12}, c_{13}, c_{14}, c_{15}, c_{16}, c_{17}, c_{18}\} = \min\{0.1, 0.3, 0.4\} = 0.1$

Similarly, we can find as $t_{13} = 0$. Thus, $d_{11} = \max\{0.1, 0.1, 0\} = 0.1$

Similarly we can find $d_{21} = 0.2$, $d_{31} = 0$. Finally, we can obtain the fs-max-min decision fs-matrix as

$$([\mathbf{a_{ij}}] \wedge [\mathbf{b_{ik}}]) \wedge [\mathbf{f_{il}}] = [\mathbf{d_{i1}}] = \begin{bmatrix} 0.1 \\ 0.2 \\ 0 \end{bmatrix}$$

Step 5:

Finally, we can find an optimum fuzzy set on U according to

$Mm(([a_{ij}] \wedge [b_{ik}]) \wedge [f_{il}])$

$opt_{Mm(([a_{ij}] \wedge [b_{ik}]) \wedge [f_{il}])}(U) = \{0.1/velachery, 0.2/ambathur, 0/porur\}$

where $Ambathur$ is an optimum area for Sales Manager , Project Manager andProduction Manager.

Similarly, we can also use products $(([a_{ij}] \vee [b_{ik}]) \vee [f_{il}])$, $(([a_{ij}] \overline{\wedge} [b_{ik}]) \overline{\wedge} [f_{il}])$, $(([a_{ij}] \underline{\vee} [b_{ik}]) \underline{\vee} [f_{il}])$ for the other convenient problems.

IV. CONCLUSIONS

In this paper we investigated the complement property for the some products of fs-matrices as well as complement for the complement of the products from product of fs-matrices.

We presented a new application with simple computational procedure which helps more than two decision makers to choose best location to build a new industry.

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