SELECTING A SUITABLE SCHOOL FOR A CHILD USING AN INTEGRATED FUZZY AHP MODEL AND ROBUST RANKING TECHNIQUE

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Abstract- Selecting a school for the children will be a little tough task for the parents as it decides the future of the child. Also there will be more criteria for the parent to select the best school for their child. In this paper we have used fuzzy AHP method combined with Robust ranking technique in triangular fuzzy numbers to find suitable school for the child which satisfies the criteria of the parents.

Key words- Fuzzy AHP, Robust Ranking technique, Triangular fuzzy numbers.

1. INTRODUCTION AND PRELIMINARIES

In day to day life people face many situations in which they have to take decision. Also they need a systematic and comprehensive approach that will provide more effective decision making support when any difficult problem arises for them. In mathematics there are many methods to solve problems that require multi-criteria decision making process such as AHP, ANP, VIKOR, PROMETHEE, ELECTRE, GRA, TOPSIS, etc.

In 1977, Saaty introduced Analytic Hierarchy Process (AHP), which is the most common multi-criteria decision making method, and now a day AHP has achieved its attraction by many researchers because of the precise mathematical properties. Also the required input data for AHP method are easy to obtain while comparing to other Multi-criteria decision making techniques. Fuzzy Analytical Hierarchy Process (FAHP) is a synthetic extension of classical Analytical Hierarchy Process. FAHP method has been effectively applied in solving various real life problems.

In (2018), Biswas, T.K., Akash S.M, Saha, used Fuzzy AHP method for selecting best apparel item to start-up with a new garment factory. In (2011), Hadad, Y., &Hanani, M.Z., combined the AHP and DEZ methodologies for selecting the best alternative. Saaty, T.L. in 1994 has given the highlights and critical points in the theory and application of the AHP method. In 1996, Chang, D.Y introduced a new approach for handling fuzzy AHP with the use of triangular fuzzy numbers for pair wise comparison scale of fuzzy AHP. WsmailiDooki, A., Bolhasani, P., &Fallah, M. in the year 2017, developed an algorithm by integrating Fuzzy AHP and Fuzzy Topsis approach for ranking and selecting the chief inspectors of banks. Iftikhar, Musheer Ahamad & Anwar

Shahzad Siddiqui, in (2017), used Fuzzy AHP method in tie breaking procedure. Rakesh Kumar Triapathi, Dookhitram. K, VivekRaich and Hana Tomskova (2013), used difference operator to solve the problem related to diabetes. Ramesh Kumar . M., Subramananian . S., (2018), solved transportation problem by using robust ranking technique. In 2020, Sophia Porchelvi, R., & Dorathy, C., solved the problem on selecting opted course for the students in HSC who completed SSLC, using integrated method of Robust ranking technique and Saaty's approach of medical diagnosis.

In this paper, Fuzzy AHP method is integrated with Robust ranking technique to select a ***School*** for the child.

Definition 1.1

A triangular fuzzy number which is denoted by $M =< m, \alpha, \beta >$, has the membership

function

$$\mu_M(x) = \begin{cases} 0 & for\ x \leq m - \alpha \\ 1 - \dfrac{m - x}{\alpha} & for\ m - \alpha < x < m \\ 1 & for\ x = m \\ 1 - \dfrac{x - m}{\beta} & for\ m < x < m + \beta \\ 0 & for\ x \geq m + \beta. \end{cases}$$

The point m with membership grade of 1, is called the mean value and α, β are the left hand and right hand spreads of M respectively. A TFN is said to be symmetric if both its spreads are equal, that is, if $\alpha = \beta$ and it is sometimes denoted by $M = \langle m, \alpha \rangle$.

Definition 1.2

If $\tilde{a}$ is a fuzzy number then the Robust ranking is defined by $R(\tilde{a}) = \int_0^1 0.5(a_\alpha^L a_\alpha^U) d\alpha$, where $(a_\alpha^L a_\alpha^U) = \{((b - a)\alpha + a, c - (c - b)\alpha)\}$. In this paper we use this method for ranking the objective values. The Roubast ranking index $R(\tilde{a})$ gives the representative value of fuzzy **number$\tilde{a}$.**

Definition 1.3

The comparison table is a square table with equal number of rows and columns where both rows and columns are labeled by the product namesP_1, P_2 ,........, P_m and the entries are 1,2,3,......, m=c_{ij},with i, j given by c_{ij} = the number of selection criteria for which the membership value of p_i exceeds or equal to the membership value of P_j .

2. ALGORITHM

Step 1: Construct a pair wise comparison triangular fuzzy matrix for the criteria.

Step 2: Calculate the value of fuzzy synthetic extent with respect to its object using, the formula

$$S_i = \sum_{j=1}^{m} M_{gi}^{j} \otimes \left[\sum_{i=1}^{n} \sum_{j=1}^{m} M_{gi}^{j}\right]^{-1} \quad (1)$$

For obtaining the value of $\sum_{gi}^{j} M_{gi}^{j}$, perform fuzzy addition operation of m extent analysis values for a particular matrix such that,

$$\sum_{j=1}^{m} M_{gi}^{j} = \left(\sum_{j=1}^{m} l_j, \sum_{j=1}^{m} m_j, \sum_{j=1}^{m} u_j\right) \quad (2)$$

Then to obtain the values of $\left[\sum_{i=1}^{n} \sum_{j=1}^{m} M_{gi}^{j}\right]^{-1}$, perform the fuzzy addition operation of $\sum M_{gi}^{j}$ values such that,

$$\left(\sum_{i=1}^{n} \sum_{j=1}^{m} M_{gi}^{j}\right) = \left(\sum_{i=1}^{n} l_j, \sum_{i=1}^{n} m_j, \sum_{i=1}^{n} u_j\right) \quad (3)$$

Then compute the inverse of the vector in equation (3) such that,

$$\left[\sum_{i=1}^{n} \sum_{j=1}^{m} M_{gi}^{j}\right]^{-1} = \left(\frac{1}{\sum_{i=1}^{n} u_i}, \frac{1}{\sum_{i=1}^{n} m_i}, \frac{1}{\sum_{i=1}^{n} l_i}\right) \quad (4)$$

Step 3: The degree of possibility of $S_2 = (l_2, m_2, u_2) \geq S_1 = (l_1, m_1, u_1)$ is defined as

$$V(S_2 \geq S_1) = sup\,[min(\mu_{S1}(X), \mu_{S2}(Y)],$$

and this can be equivalently expressed as,

$$V(S_2 \geq S_1) = highest(S_1 \cap S_2) = \mu_{S2}(d),$$

$$= \begin{Bmatrix} 1 & if\, m_2 \geq m_1 \\ 0 & if\, l_1 \geq u_2 \\ \dfrac{l_1 - u_2}{(m_2 - u_2) - (m_1 - l_1)} & Otherwise \end{Bmatrix}$$

where d is the ordinate of the highest intersection point D between μ_{M1} and μ_{M2}, alsoto compare M_1 and M_2 we need both the values of $V(S_1 \geq S_2)$ and $(S_2 \geq S_1)$.

Step 4: The degree of possibility for a convex fuzzy number to be larger than k convex fuzzy numbers $S_i\ (i = 1,2,3, \dots . k)$ can be defined by,

$$V(S \geq S_1, S_2, \dots S_k) = V[(S \geq S_1) \cap (S \geq S_2) \cap \dots (S \geq S_k)]$$
$$= min\, V(S \geq S_i), i = 1,2,3, \dots k.$$

Let us assume that $d'(A_i) = min\, V(S_i \geq S_k). For\ k = 1,2, \dots n;\ k \neq i$. Then the weight vector is given by μ and $W' = (d'(A_1), d'(A_2) \dots . d'(A_n))^T$, where $A_i (i = 1,2, \dots . n)$ are n elements.

Step 5: Via normalization, the normalized weight vectors

are $W = (d(A_1), d(A_2), \ldots . d(A_n))^T$

where W is a non-fuzzy number. That gives the priority weights of one alternative over another.

Step 6: Construct a triangular fuzzy matrix A using the criteria and alternatives.

Step 7: Convert the triplet values of A into normal fuzzy numbers using the robust ranking technique.

Step 8: Multiply the matrix A we obtained in step 7 with the weight W we obtained in step 5 which is known as a comprehensive decision matrix D.

Step 9: Construct comparison table for the comprehensive decision matrix D.

Step 10: Find the row sum and column sum for the comparison table.

Step 11: Find the difference between the row sum and column sum in the comparison table, and the alternative with maximum score is recommended as the best choice.

3. PROBLEM

For every parent, selecting a suitable school for their child will be a difficult job, as there will be many schools and they have to select a suitable school for their child which also satisfies their criteria. Here an attempt has been made to select a suitable school for a child. We have selected five criteria and we have compared three schools and from that we have given suitable school for the child which also satisfied the criteria of the parents and their expectation.

Here we are taking criteria as

$\boldsymbol{C_1 : Vision\ and\ Mission}$ $\boldsymbol{C_2 : Curriculum}$ $\boldsymbol{C_3 : Faculty}$

$\boldsymbol{C_4 : Infrastructure}$ $\boldsymbol{C_5 : Extra - Curricular\ activity .}$

And schools as

$\boldsymbol{S_1 : School\ 1}$; $\boldsymbol{S_2 : School\ 2;}$ $\boldsymbol{S_3 : School\ 3.}$

For this, first we use AHP method to find the weight of criteria. For that we constructed a fuzzy linguistic table,

Linguistic Variables	***TriangularFuzzy Numbers***
Equally depends	(1,1,1)
Very strongly depends	(0.8,0.9,1)
Strongly depends	(0.7,0.85,1)
Moderately depends	(0.6,0.65,0.7)
Weakly depends	(0.5,0.6,0.7)

Construct a fuzzy triangular pair wise comparison matrix using the criteria.

	C_1	C_2	C_3	C_4	C_5
C_1	(1,1,1)	(0.7,0.85,1)	(0.5,0.6,0.7)	(0.6,0.65,0.7)	(0.8,0.9,1)
C_2	(0.8,0.9,1)	(1,1,1)	(0.5,0.6,0.7)	(0.5,0.6,0.7)	(0.8,0.9,1)
C_3	(0.5,0.6,0.7)	(0.5,0.6,0.7)	(1,1,1)	(0.5,0.6,0.7)	(0.5,0.6,0.7)
C_4	(0.5,0.6,0.7)	(0.6,0.65,0.7)	(0.6,0.65,0.7)	(1,1,1)	(0.7,0.85,1)
C_5	(0.6,0.65,0.7)	(0.6,0.65,0.7)	(0.5,0.6,0.7)	(0.7,0.85,1)	(1,1,1)

Value of fuzzy synthetic extent with respect to its object is calculated using the formulas we have given in procedure.

$$\sum_{i=1}^{n}\sum_{j=1}^{n} M_{ij} = (17,18.90,20.80)$$

Inverse of the vectors be, $\left(\sum_{i=1}^{n}\sum_{j=1}^{n} M_{ij}\right)^{-1} = (\frac{1}{20.80}, \frac{1}{18.90}, \frac{1}{17})$

$$S_{C_1} = (3.6,4,4.4) \times (\frac{1}{20.80}, \frac{1}{18.90}, \frac{1}{17})$$

$$S_{C_1} = (0.17,0.212,0.26)$$

$$S_{C_2} = (3.6,4,4.4) \times (\frac{1}{20.80}, \frac{1}{18.90}, \frac{1}{17})$$

$$S_{C_2} = (0.17,0.212,0.26)$$

$$S_{C_3} = (3,3.4,3.8) \times (\frac{1}{20.80}, \frac{1}{18.90}, \frac{1}{17})$$

$$S_{C_3} = (0.14,0.18,0.22)$$

$$S_{C_4} = (3.4,3.75,4.1) \times (\frac{1}{20.80}, \frac{1}{18.90}, \frac{1}{17})$$

$$S_{C_4} = (0.16,0.20,0.24)$$

$$S_{C_5} = (3.4,3.75,4.1) \times (\frac{1}{20.80}, \frac{1}{18.90}, \frac{1}{17})$$

$$S_{C_5} = (0.16,0.20,0.24)$$

Now calculate the degree of possibility using the formula we have given in step 3.

$V(S_{C_1} \geq S_{C_2}) = 1$ $V(S_{C_2} \geq S_{C_1}) = 1$ $V(S_{C_3} \geq S_{C_1}) = 0.63$

$V(S_{C_1} \geq S_{C_3}) = 1$ $V(S_{C_2} \geq S_{C_3}) = 1$ $V(S_{C_3} \geq S_{C_2}) = 0.63$

$V(S_{C_1} \geq S_{C_4}) = 1$ $V(S_{C_2} \geq S_{C_4}) = 1$ $V(S_{C_3} \geq S_{C_4}) = 0.75$

$V(S_{C_1} \geq S_{C_5}) = 1$ $V(S_{C_2} \geq S_{C_5}) = 1$ $V(S_{C_3} \geq S_{C_5}) = 0.75$

$V(S_{C_4} \geq S_{C_1}) = 0.88$ $V(S_{C_5} \geq S_{C_1}) = 0.88$

$V(S_{C_4} \geq S_{C_2}) = 0.88$ $V(S_{C_5} \geq S_{C_2}) = 0.88$

$V(S_{C_4} \geq S_{C_3}) = 1$ $V(S_{C_5} \geq S_{C_3}) = 1$

$V(S_{C_4} \geq S_{C_5}) = 1$ $V(S_{C_5} \geq S_{C_4}) = 1$

Using step. 4 to calculate the degree of possibility for a convex fuzzy number to be larger than K convex fuzzy numberM_i, let it be,

$d'(C_1) = \min(1,1,1,1)$ $\Rightarrow d'(C_1) = 1$

$d'(C_2) = \min(1,1,1,1)$ $\Rightarrow d'(C_2) = 1$

$d'(C_3) = \min(0.63,0.63,0.75,0.75) \qquad \Rightarrow d'(C_3) = 0.63$

$d'(C_4) = \min\ (0.88,0.88,1,1) \qquad \Rightarrow d'(C_4) = 0.88$

$d'(C_5) = \min\ (0.88,0.88,1,1) \qquad \Rightarrow d'(C_5) = 0.88$

The normalized weight vectors are calculated using the formula in step.5,

$$W' = (1,1,0.63,0.88,0.88)^T$$

$$\sum W' = 4.39$$

$$W = (0.23,0.23,0.14,0.20,0.20)^T$$

Now construct a linguistic variable for comparing criteria and alternatives,

Linguistic variable	*Triangular fuzzy number*
Extremely strong	(0.9,0.9,0.9)
Verry Strong	(0.8,0.85,0.9)
Strong	(0.7,0.8,0.9)
Weakly strong	(0.6,0.7,0.8)
Very	(0.5,0.6,0.7)

Construct a triangular fuzzy matrix using the criteria and alternatives,

$$A = \begin{array}{c} \\ S_1 \\ S_2 \\ S_3 \end{array} \begin{array}{c} \begin{array}{ccccc} C_1 & C_2 & C_3 & C_4 & C_5 \end{array} \\ \begin{bmatrix} (0.9,0.9,0.9) & (0.8,0.85,0.9) & (0.7,0.8,0.9) & (0.5,0.6,0.7) & (0.5,0.6,0.7) \\ (0.9,0.9,0.9) & (0.7,0.8,0.9) & (0.6,0.7,0.8) & (0.6,0.7,0.8) & (0.6,0.7,0.8) \\ (0.7,0.8,0.9) & (0.7,0.8,0.9) & (0.5,0.6,0.7) & (0.7,0.8,0.9) & (0.8,0.85,0.) \end{bmatrix} \end{array}$$

Using robust ranking technique to convert the triplet values in to single values, and the resultant matrix is,

$$A = \begin{array}{c} \\ S_1 \\ S_2 \\ S_3 \end{array} \begin{array}{c} \begin{matrix} C_1 & C_2 & C_3 & C_4 & C_5 \end{matrix} \\ \begin{bmatrix} 0.9 & 0.85 & 0.8 & 0.6 & 0.6 \\ 0.9 & 0.8 & 0.7 & 0.7 & 0.7 \\ 0.8 & 0.8 & 0.6 & 0.8 & 0.85 \end{bmatrix} \end{array}$$

Multiplying matrix A with the weight W and the resulting comprehensive decision matrix is given as,

$$D = \begin{array}{c} \\ S_1 \\ S_2 \\ S_3 \end{array} \begin{array}{c} \begin{matrix} C_1 & C_2 & C_3 & C_4 & C_5 \end{matrix} \\ \begin{bmatrix} 0.210 & 0.200 & 0.110 & 0.112 & 0.120 \\ 0.210 & 0.180 & 0.100 & 0.140 & 0.140 \\ 0.180 & 0.180 & 0.080 & 0.160 & 0.170 \end{bmatrix} \end{array}$$

Construct a comparison table

	S_1	S_2	S_3
S_1	3	3	3
S_2	3	3	3
S_3	2	3	3

COMPARISON TABLE

Now, compute the row-sum, column-sum from the comparison table and the score for each S_i is also calculated and is given below:

	Row sum	*Column sum*	*Score*
S_1	9	8	**1**
S_2	9	9	0
S_3	8	9	-1

4. Conclusion

From the table above, it is clear that school S_1 will be the very suitable school for the parent as it satisfies all the criteria of them to the fullest Also the method and the algorithm we used to solve this problem is easy to understand and to solve.

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